CONDITIONAL CORRELATION
AND CONDITIONAL VOLATILITY

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Introduction (1)

- stock market linkages have been attracting the attention of analysts for a long time
- there exist plenty of studies dealing with this issue using various ways and methods of analysis in order to capture how shocks from one market can be transmitted to another market(s)
- it is also known that the correlation of emerging markets with the developed markets is relatively low and returns in emerging markets are much higher than in the developed markets ⇒ opportunities for international diversification
- to analyse the co-movements of financial returns from different markets, especially to study volatility spillover, i.e. if and how the shocks from one stock market influence the volatility development of the other market is an interesting and challenging issue
Introduction (2)

- very interesting issue is also to study the relationships between the conditional correlation and conditional volatility
- Cappiello et al. (2006) and Gjika and Horvath (2012) pointed out the fact and proved that the stock market conditional volatility and correlation are positively related
- **Aim of the presentation**: to analyse the relationships between the dynamic conditional correlation and conditional volatility for the selected CEE markets (Czech Republic, Hungary, Poland) and the Western European stock market represented by stock market of Germany and France, respectively, as well as the relationship between German and French market and between individual CEE markets for weekly data during the period 2003-2013; the impact of the higher volatility during the period October 17th, 2008 – December 25th, 2009 is also analyzed
Data (1)

- analyzed data set: weekly data of stock price indices of CEE countries - the Czech PX, Hungarian BUX, Polish WIG20 and two Western European stock indices - the German DAX and French CAC40
- data source: www.stooq.com
- if we assume that $P_t$ is the closing value of the stock index at time $t$ and $r_t$ denotes logarithm of the corresponding stock return, the formula for calculation of the logarithmic stock return is as follows:

$$r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \times 100\%$$
<table>
<thead>
<tr>
<th></th>
<th>DLBUX</th>
<th>DLPX</th>
<th>DLWIG</th>
<th>DLDAX</th>
<th>DLCAC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.151860</td>
<td>0.132428</td>
<td>0.119699</td>
<td>0.197812</td>
<td>0.051615</td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td>3.562397</td>
<td>3.311555</td>
<td>3.247232</td>
<td>3.207019</td>
<td>3.043438</td>
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<tr>
<td><strong>Skewness</strong></td>
<td>-0.978616</td>
<td>-1.549669</td>
<td>-0.397702</td>
<td>-0.953029</td>
<td>-1.293369</td>
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<tr>
<td><strong>Kurtosis</strong></td>
<td>10.21027</td>
<td>17.57873</td>
<td>5.867259</td>
<td>10.93746</td>
<td>12.19645</td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>1332.674***</td>
<td>5303.720***</td>
<td>211.3854***</td>
<td>1590.943***</td>
<td>2178.974***</td>
</tr>
</tbody>
</table>
Methodological Issues (1)

• conditional mean equation

\[ r_t = \omega_0 + \varepsilon_t \]

• conditional volatility – GARCH model

\[ h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \]

• dynamic conditional correlation (DCC) – multivariate ARCH-class model
  – estimation follows in two steps:
    1) estimation of univariate ARCH-class models
    2) estimation of conditional correlation coefficients’ matrix
Methodological Issues (2)

- DCC model
  
  $k \times 1$ dimensional vector of stock returns $\mathbf{r}_t$
  
  $\mathbf{r}_t \mid \Omega_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t)$
  
  conditional variance-covariance matrix $\mathbf{H}_t$
  
  \[ \mathbf{H}_t = \mathbf{D}_t \mathbf{C}_t \mathbf{D}_t \]
  
  $\mathbf{D}_t$ is the $k \times k$ diagonal matrix
  with the time-varying standard deviations from univariate GARCH models on the diagonal
  
  $\mathbf{C}_t$ is the time-varying correlation matrix
Methodological Issues (3)

\[ C_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2} \]

\[ Q_t = (1 - q_a - q_b) \bar{Q} + q_a z_{t-1} z_{t-1}^T + q_b Q_{t-1} \]

\[ \bar{Q} = E\left( z_t z_t^T \right) \quad q_a + q_b < 1 \]

\[ \rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} ; \quad i, j = 1, 2, \ldots, n; \quad i \neq j \]
Empirical Results – Conditional Volatilities

GARCH_BUX

GARCH_PX

GARCH_WIG

GARCH_DAX

GARCH_CAC
Empirical Results - Unconditional Correlations

- unconditional correlations between all pairs of stock returns

<table>
<thead>
<tr>
<th></th>
<th>DLBUX</th>
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<th>DLWIG</th>
<th>DLDAX</th>
<th>DLCAC</th>
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</thead>
<tbody>
<tr>
<td>DLBUX</td>
<td>1,000000</td>
<td>0,714319</td>
<td>0,676988</td>
<td>0,593688</td>
<td>0,620141</td>
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<td>1,000000</td>
<td>0,682225</td>
<td>0,645627</td>
<td>0,676240</td>
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<tr>
<td>DLWIG</td>
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<td>1,000000</td>
<td>1,000000</td>
<td>0,609298</td>
<td>0,598369</td>
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<td>DLDAX</td>
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<tr>
<td>DLCAC</td>
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Empirical Results – Dynamic Conditional Correlations (1)

DCC_BUX_DAX

DCC_PX_DAX

DCC_WIG_DAX

DCC_BUX_CAC

DCC_PX_CAC

DCC_WIG_CAC
Empirical Results – Dynamic Conditional Correlations (2)

DCC_CAC_DAX

DCC_BUX_PX

DCC_BUX_WIG

DCC_PX_WIG
DCC and Conditional Volatility (1)

• relationship between conditional correlations and conditional volatilities

\[ \rho_{ij,t} = \omega + \beta_i h_{i,t} + \beta_j h_{j,t} + \epsilon_{ij,t} \]

• \( \rho_{ij,t} \) is the estimated pair-wise conditional correlation coefficient between the stock returns of markets \( i \) and \( j \)

• positive value of \( \beta_i \) indicates that conditional correlations between the market \( i \) and \( j \) rise with the volatility of the market \( i \), whereas negative value of \( \beta_i \) means that the correlations between the return series of market \( i \) and market \( j \) fall in periods of high volatility in the market \( i \)
DCC and Conditional Volatility (2)

- relationship between conditional correlations and conditional volatilities concerning the dummy variable $dum_t$ taking the value of 1 in the period of higher conditional volatility (October 17th, 2008 – December 25th, 2009)

$$\rho_{ij,t} = \omega + \beta_i h_{i,t} + \beta_j h_{j,t} + \alpha_0 dum_t + \alpha_i dum_t h_{i,t} + \alpha_j dum_t h_{j,t} + \varepsilon_{ij,t}$$

- where $\alpha_0$ denotes the shift in the level of DCC during the above mentioned period, positive values of $\alpha_i$ and $\alpha_j$ indicate the rise of conditional correlations between the market $i$ and $j$ with the rise of volatility of the market $i$ and $j$, respectively during the period October 17th, 2008 – December 25th, 2009
Empirical Results - DCC and Conditional Volatility

\[ \rho_{ij,t} = \omega + \beta_i h_{i,t} + \beta_j h_{j,t} + \alpha_0 \text{dum}_t + \alpha_i \text{dum}_t h_{i,t} + \alpha_j \text{dum}_t h_{j,t} + \varepsilon_{ij,t} \]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(j)</th>
<th>BUX DAX</th>
<th>PX DAX</th>
<th>WIG DAX</th>
<th>CAC DAX</th>
<th>BUX CAC</th>
<th>PX CAC</th>
<th>WIG CAC</th>
<th>BUX PX</th>
<th>BUX WIG</th>
<th>PX WIG</th>
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<tbody>
<tr>
<td>(\omega)</td>
<td>0,486</td>
<td>0,532</td>
<td>0,594</td>
<td>0,899</td>
<td>0,505</td>
<td>0,578</td>
<td>0,578</td>
<td>0,634</td>
<td>0,543</td>
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<tr>
<td>(\beta_i)</td>
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<td>4.10^{-4}</td>
<td>-0,001</td>
<td>-0,001</td>
<td>-3.10^{-4}</td>
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<td>0,006</td>
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<tr>
<td>(\beta_j)</td>
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<td>0,001</td>
<td>2.10^{-4}</td>
<td>0,009</td>
<td>0,007</td>
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<td>0,004</td>
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<tr>
<td>(\alpha_0)</td>
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<td>0,006</td>
<td>0,185</td>
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<td>0,003</td>
<td>-4.10^{-4}</td>
<td>0,002</td>
<td>0,001</td>
<td>2.10^{-4}</td>
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<td>-0,002</td>
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<tr>
<td>(\alpha_j)</td>
<td>-0,001</td>
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<td>1.10^{-4}</td>
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<td>-0,001</td>
<td>0,001</td>
<td>-0,004</td>
<td>-0,002</td>
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</table>
Conclusion

• concerning the statistical significance of the estimated parameters, the DCC of individual CEE return series with the Western European return series (DAX and CAC40) rise as a result of growing volatility of Western European returns (positive values of $\beta_j$), whereas during the period October 17th, 2008 – December 25th, 2009 the decline in DCC was recorded as a reaction to the rising volatility of DAX and CAC40, respectively (negative values of $\alpha_j$)
• the impact of volatilities in CEE countries on DCC values was not so clear (statistically significant positive and negative values of $\beta_i$ and $\alpha_i$ as well as in some cases non-significant parameters’ values)
• no impact of volatility on conditional correlation was detected in case of DCC_DAX_CAC
• not so clear results were received for the pairs of CEE countries’ returns
References


• http://forums.eviews.com/

• http://stooq.com