CVaR
as linear programming model

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Conditional value at risk (CVaR)

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Conditional value at risk (CVaR)

- Value at risk (VaR) a standard tool of risk management in the financial sector
- An alternative measure of risk—conditional VaR
- Shortcomings of VaR:
  I. VaR measures only percentiles of profit and loss, and thus disregards the loss beyond the VaR,
  II. VaR is not a coherent risk measure because it is not sub-additive
CVaR as linear programming model

Model of portfolio selection based on CVaR:

\[ CVaR_\alpha(X) = \min \left\{ Var_\alpha + \frac{1}{\alpha} E \left[ (E_p - X - Var_\alpha)^+ \right] \right\} \]

where \( Var_\alpha \) – Value at risk, \( E_p \) – target return, formula \((E_p - X - Var)^+\) is a positive part of difference \(E_p - X - Var\).
CVaR as linear programming model

Variable $X$ – expectant return of portfolio $w^T r_k$

Objective function:

$$\min \left\{ \text{VaR}_\alpha + \frac{1}{\alpha t} \sum_{k=1}^{z} [E_p - w^T r_k - \text{VaR}_\alpha]^+] \right\}$$
To avoid the nonlinear formulation it is necessary to replace the element

\[
[E_p - w^T r_k - \text{VaR}_\alpha]^+
\]

with the variable \( z = (z_1, z_2, ..., z_t) \), where \( z_k \geq 0 \) for \( k=1,2,\ldots,t \).
CVaR as linear programming model

\[
\min \left\{ \text{VaR}_\alpha + \frac{1}{\alpha t} \sum_{k=1}^{z} z_k \right\}
\]

\[
z_k - E_p + w^T r_k + \text{VaR}_\alpha \geq 0, \quad k = \{1, 2, \ldots, t\},
\]

\[
w^T E(r_n) \geq E_p
\]

\[
w^T e = 1
\]

\[
z \geq 0,
\]

\(E(r_n)\) – vector of expected returns of assets