Econometric Modelling of Financial Time Series: Selected Approaches

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The main aim of the presentation

- a brief introduction into the econometric modelling of the financial time series

2 parts:

- volatility modelling using the ARCH-class models
- investigation of the relationships between pairs of time series based on correlation analysis, cointegration concept and Granger causality with illustration for the Netherland stock exchange data
Introduction (ARCH-class Models)

- financial time series are mostly analysed in the form of returns (first differences), the main feature of which is the time-varying volatility
- conditional volatility approach - Engle (1982), Bollerslev (1986), etc.
- today - huge amount of various ARCH-class models
- from the perspective of the rational investor represents the height of returns only one part of the decision making process
- another essential factor to take into account is the amount of risk, i.e. the volatility of returns together with its seasonal anomalies (turn-of-the-year, turn-of-the-month, day-of-the-week effects, etc.)
- interesting area is also to investigate the role of the trading volume in explanation of the volatility persistence of stock returns
Data and Methodology

- the analysis is usually done on logarithmic transformation of return series
- the logarithmic return series are also calculated as the logarithmic first differences of the individual series measured in levels, e.g. daily closing values of the stock indices or daily exchange rates:

\[ r_t = d(\ln(P_t)) = \ln\left(\frac{P_t}{P_{t-1}}\right) \]

where \( P_t \) is the „price“ (e.g. closing value of the stock index, exchange rate) at time \( t \) and \( r_t \) denotes logarithm of the corresponding return
Methodological aspects:

- the logarithmic returns equation, i.e. the conditional mean equation - Box-Jenkins ARMA(m,n) model
- conditional variance equation - appropriate ARCH-class model
### Some ARCH-class Models

<table>
<thead>
<tr>
<th>Model/ authors, year</th>
<th>Mathematical form/ parameter conditions/some explanations</th>
</tr>
</thead>
</table>
| **ARCH(q)** Engle, 1982 | $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2$  
$\alpha_0 > 0$, $\alpha_i \geq 0$ for $i=1,2,\ldots,q$ |
| **GARCH(p,q)** Bollerslev, 1986 | $h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$  
$\alpha_0 \geq 0$, $\alpha_i \geq 0$ for $i=1,2,\ldots,q$, $\beta_j \geq 0$ for $j=1,2,\ldots,p$ |
| **IGARCH(p,q)** Engle and Bollerslev, 1986 | $h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$ for $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1$ |
| **EGARCH(p,q,r)** Nelson, 1991 | $\ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sqrt{h_{t-k}}}$  
leverage effect: $\gamma_k < 0$, asymmetry in volatility: $\gamma_k \neq 0$ |
| **TGARCH(p,q)** Zakoian, 1990 | $h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i} I_{t-i}$  
$\alpha_0 > 0$, $\alpha_i \geq 0$, $\alpha_i + \gamma_i \geq 0$ for $i=1,2,\ldots,q$, $\beta_j \geq 0$ for $j=1,2,\ldots,p$  
$I_{t-i} = \begin{cases} 1, & \text{if } \varepsilon_{t-i} < 0 \\ 0, & \text{if } \varepsilon_{t-i} > 0 \end{cases}$  
leverage effect: $\gamma_i > 0$, asymmetry in volatility: $\gamma_i \neq 0$ |
| **PARCH(p,q)** Ding et al., 1993 | $\left(\sqrt{h_t}\right)^\delta = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\left|\varepsilon_{t-i}\right| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{p} \beta_j \left(\sqrt{h_{t-j}}\right)^\delta$  
$\alpha_0 > 0, \delta \geq 0, \beta_j \geq 0$ for $j=1,2,\ldots,p$, $\alpha_i \geq 0$ for $i=1,2,\ldots,q$  
asymmetry in volatility: $\gamma_i \neq 0$ |
Modelling of seasonal anomalies:
a GARCH(p,q) case study

- Conditional mean equation

\[ r_t = \omega_0 + \sum_{j=1}^{m} \phi_j r_{t-j} + \varepsilon_t + \sum_{k=1}^{n} \theta_k \varepsilon_{t-k} + \sum_{l=2}^{12} K_l Y_{lt} + \Lambda M_t + \sum_{r=2}^{5} \Pi_r D_{rt} \]

- Conditional variance equation

\[ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{l=2}^{12} \kappa_l Y_{lt} + \Lambda M_t + \sum_{r=2}^{5} \pi_r D_{rt} \]

*turn-of-the-year effect*: dummy variables \( Y_{lt} \ (l = 2,3,...,12) \) representing months from February to December

*turn-of-the-month effect*: dummy variable \( M_t \) taking the value of one in the month’s first fifteen calendar days (and zero in the remaining days)

*day-of-the-week effect*: dummy variables \( D_{rt} \ (r = 2,3,4,5) \) for individual trading days with exception of Monday
The role of the trading volume in explanation of the volatility persistence of stock returns

GARCH(p,q) case study

- to examine the effect of trading volume on stock returns volatility, the following modification of the conditional variance equation is used

\[ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \delta V_t \]

where \( V_t \) is the logarithm of the trading volume

- according to the Lamoureux et al. (1990) the parameter \( \delta \) should be positive and the volatility persistence should become negligible
Interactions between the Financial Time Series

- e.g. mutual relationships between:
  - exchange rates and stock indices
  - pairs of exchange rates
  - pairs of stock indices

- various procedures:
  - graphical analysis
  - correlation procedures
  - long-run relationships - cointegration procedures (Engle-Granger, Johansen)
  - short-run relationships - Granger causality, impulse responses analysis
Long-run relationships

- Cointegration procedures:
  - Engle-Granger = cointegration approach for a bivariate system (one cointegrating vector)
    - based on testing of residuals from cointegrating equation performing e.g. the ADF test on residuals
  - Johansen = more general technique applicable also in case of more than two variables (for $N$ number of variables we can have up to $N-1$ cointegrating vectors)
    - based on maximum likelihood method using both the $\lambda_{\text{trace}}$ and $\lambda_{\text{max}}$ statistics
Short-run relationships

Granger causality

-we can say, that the time series $x_t$ Granger-causes time series $y_t$, if $y_t$ can be predicted better by using past values of $x_t$ than by using only the historical values of the $y_t$
-if this doesn’t hold, we can say that $x_t$ doesn’t Granger-cause $y_t$
-whether the time series $y_t$ Granger-causes the time series $x_t$ can be tested in analogical way
-the corresponding VAR($k$) model:

$$y_t = \alpha_{10} + \sum_{j=1}^{k} \alpha_{1j}x_{t-j} + \sum_{j=1}^{k} \beta_{1j}y_{t-j} + \varepsilon_{1t}$$

$$x_t = \alpha_{20} + \sum_{j=1}^{k} \alpha_{2j}x_{t-j} + \sum_{j=1}^{k} \beta_{2j}y_{t-j} + \varepsilon_{2t}$$

-testing statistics: Wald F-statistics

Impulse responses analysis

-to examine the short-run dynamic relations between analysed time series
Relationships between the overall stock market index and its sectoral indices:
A Case Study for Netherland

Graphical analysis

**general index:** AEX

**sectoral indices:**
Basic Materials (BAS_MA),
Consumer Goods (CON_GO),
Consumer Services (CON_SER),
Financials (FIN), Health Care (HC),
Industrials (INDU), Oil&Gas (OIG),
Telecommunications (TEL), Technology (TECH)

source of data: NYSE Euronext webpage

*Note: prefix “L“ denotes natural logarithm*
## Descriptive statistics and ADF test results - indices

<table>
<thead>
<tr>
<th>Indices</th>
<th>LAEX</th>
<th>LBAS_MA</th>
<th>LCON_GO</th>
<th>LCON_SER</th>
<th>LFIN</th>
<th>LHC</th>
<th>LINDU</th>
<th>LOIG</th>
<th>LTECH</th>
<th>LTEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev.</td>
<td>0.0793</td>
<td>0.2063</td>
<td>0.0481</td>
<td>0.0809</td>
<td>0.1282</td>
<td>0.1519</td>
<td>0.1430</td>
<td>0.0840</td>
<td>0.1080</td>
<td>0.1078</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.7260</td>
<td>-0.8168</td>
<td>-0.9615</td>
<td>-0.9383</td>
<td>-0.5488</td>
<td>0.2199</td>
<td>-0.7487</td>
<td>-0.0967</td>
<td>0.1317</td>
<td>-0.7263</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.8608</td>
<td>2.3760</td>
<td>3.4727</td>
<td>2.6388</td>
<td>2.3292</td>
<td>1.4261</td>
<td>2.3029</td>
<td>1.9028</td>
<td>2.0779</td>
<td>2.7439</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>45.7***</td>
<td>65.6***</td>
<td>84.1***</td>
<td>78.4***</td>
<td>35.5***</td>
<td>57.3***</td>
<td>58.5***</td>
<td>26.6***</td>
<td>19.7***</td>
<td>46.7***</td>
</tr>
<tr>
<td>ADF stat. (without trend and without constant)</td>
<td>-0.0207</td>
<td>-0.6035</td>
<td>-0.0463</td>
<td>-0.0442</td>
<td>-0.2334</td>
<td>0.3981</td>
<td>-0.0146</td>
<td>0.8111</td>
<td>0.6651</td>
<td>-1.3302</td>
</tr>
</tbody>
</table>

*Note: Symbols ***, **, * denote in the whole presentation the rejection of the null hypothesis on the 1 %, 5 %, and 10 % significance level, respectively.*
## Descriptive statistics and ADF test results - returns

<table>
<thead>
<tr>
<th>Returns</th>
<th>DLAEX</th>
<th>DLABAS_MA</th>
<th>DLCON_GO</th>
<th>DLCON_SER</th>
<th>DLFIN</th>
<th>DLHC</th>
<th>DLINDU</th>
<th>DLOIG</th>
<th>DLTECH</th>
<th>DLTEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0000</td>
<td>-0.0006</td>
<td>0.0000</td>
<td>-0.0002</td>
<td>0.0004</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.0005</td>
<td>-0.0007</td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0707</td>
<td>0.0852</td>
<td>0.0607</td>
<td>0.0396</td>
<td>0.1410</td>
<td>0.3021</td>
<td>0.1031</td>
<td>0.0400</td>
<td>0.0548</td>
<td>0.0366</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0457</td>
<td>-0.0840</td>
<td>-0.0393</td>
<td>-0.0463</td>
<td>-0.0886</td>
<td>-0.0997</td>
<td>-0.0760</td>
<td>-0.0493</td>
<td>-0.0536</td>
<td>-0.0862</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0134</td>
<td>0.0219</td>
<td>0.0115</td>
<td>0.0100</td>
<td>0.0215</td>
<td>0.0189</td>
<td>0.0165</td>
<td>0.0125</td>
<td>0.0165</td>
<td>0.0126</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.0207</td>
<td>0.0217</td>
<td>0.0944</td>
<td>-0.3599</td>
<td>0.3247</td>
<td>7.5419</td>
<td>0.1479</td>
<td>-0.4060</td>
<td>-0.0161</td>
<td>-1.0377</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>128.8***</td>
<td>89.4***</td>
<td>72.0***</td>
<td>119.9***</td>
<td>469.1***</td>
<td>351210.0***</td>
<td>461.1***</td>
<td>76.9***</td>
<td>15.7***</td>
<td>924.3***</td>
</tr>
</tbody>
</table>
Correlation coefficients of AEX index with its sectoral indices

- based on the graphical analysis we can expect that there are some relationships between the overall index (AEX) and some of its sectoral indices which was proved by calculation of correlation coefficients:

<table>
<thead>
<tr>
<th></th>
<th>LBAS_MA</th>
<th>LCON_GO</th>
<th>LCON_SER</th>
<th>LFIN</th>
<th>LHC</th>
<th>LINDU</th>
<th>LOIG</th>
<th>LTECH</th>
<th>LTEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>levels</td>
<td>0.9244</td>
<td>0.8484</td>
<td>0.9171</td>
<td>0.9393</td>
<td>0.7535</td>
<td>0.9440</td>
<td>0.2013</td>
<td>0.6142</td>
<td>0.6516</td>
</tr>
<tr>
<td>returns</td>
<td>0.8948</td>
<td>0.8488</td>
<td>0.8446</td>
<td>0.9249</td>
<td>0.3948</td>
<td>0.8803</td>
<td>0.7876</td>
<td>0.7601</td>
<td>0.5056</td>
</tr>
</tbody>
</table>
Engle-Granger cointegration test – residuals ADF test statistics

<table>
<thead>
<tr>
<th></th>
<th>LBAS_MA</th>
<th>LCON_GO</th>
<th>LCON_SER</th>
<th>LFIN</th>
<th>LHC</th>
<th>LINDU</th>
<th>LOIG</th>
<th>LTECH</th>
<th>LTEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF stat.</td>
<td>-2.48018</td>
<td>-3.02735</td>
<td>-2.33871</td>
<td>-2.08952</td>
<td>-2.45456</td>
<td>-2.75117</td>
<td>-1.78739</td>
<td>-0.87917</td>
<td>-0.78831</td>
</tr>
</tbody>
</table>

➢ it was not possible to reject the unit root hypothesis, i.e. no long-run relationship was confirmed
Granger causality test – results
H0: AEX index returns does not Granger cause sectoral index returns

<table>
<thead>
<tr>
<th></th>
<th>Lags: 1</th>
<th>Lags: 2</th>
<th>Lags: 3</th>
<th>Lags: 4</th>
<th>Lags: 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLBAS_MA</td>
<td>0.2579</td>
<td>2.6568*</td>
<td>2.3116*</td>
<td>2.2241*</td>
<td>1.7889</td>
</tr>
<tr>
<td>DLCON_GO</td>
<td>0.2134</td>
<td>0.7560</td>
<td>0.8691</td>
<td>0.6923</td>
<td>0.6442</td>
</tr>
<tr>
<td>DLCON_SER</td>
<td>9.3063***</td>
<td>4.8284***</td>
<td>3.2660**</td>
<td>2.5504**</td>
<td>2.1158*</td>
</tr>
<tr>
<td>DLFIN</td>
<td>1.2918</td>
<td>0.6601</td>
<td>0.5325</td>
<td>0.5528</td>
<td>0.5566</td>
</tr>
<tr>
<td>DLHC</td>
<td>2.5155</td>
<td>1.1845</td>
<td>0.8099</td>
<td>0.6098</td>
<td>0.5093</td>
</tr>
<tr>
<td>DLINDU</td>
<td>3.2203*</td>
<td>3.4277**</td>
<td>2.5070*</td>
<td>2.2781*</td>
<td>3.2654***</td>
</tr>
<tr>
<td>DLOIG</td>
<td>0.0686</td>
<td>0.0370</td>
<td>0.3193</td>
<td>0.3748</td>
<td>0.5706</td>
</tr>
<tr>
<td>DLTECH</td>
<td>0.1174</td>
<td>0.2553</td>
<td>1.3916</td>
<td>1.0361</td>
<td>1.4629</td>
</tr>
<tr>
<td>DLTEL</td>
<td>0.0332</td>
<td>0.8665</td>
<td>1.7047</td>
<td>1.4140</td>
<td>1.1833</td>
</tr>
</tbody>
</table>
A Case Study Conclusion

- it was not possible to reject the hypothesis about the existence of the unit root for the overall AEX index as well as for all the sectoral indices; the returns were stationary
- concerning the graphical analysis and correlation coefficients, the strongest relationships (correlation coefficient more than 0.9) were identified between LAEX and following sectoral indices: LBAS_MA, LCON_SER, LFIN, LINDU
- based on Granger causality test results, it was proved that for pairs with confirmed short-run relationships the correlation coefficients were high
- no long-run relationship was confirmed
- similarly as Waściński et al. (2009) we proved the existence of the short-run relationships in some cases (although the results are strongly dependent on the concrete analysed pair of indices), but the long-run relationships of the overall AEX index to its sectoral indices were confirmed in neither case
References