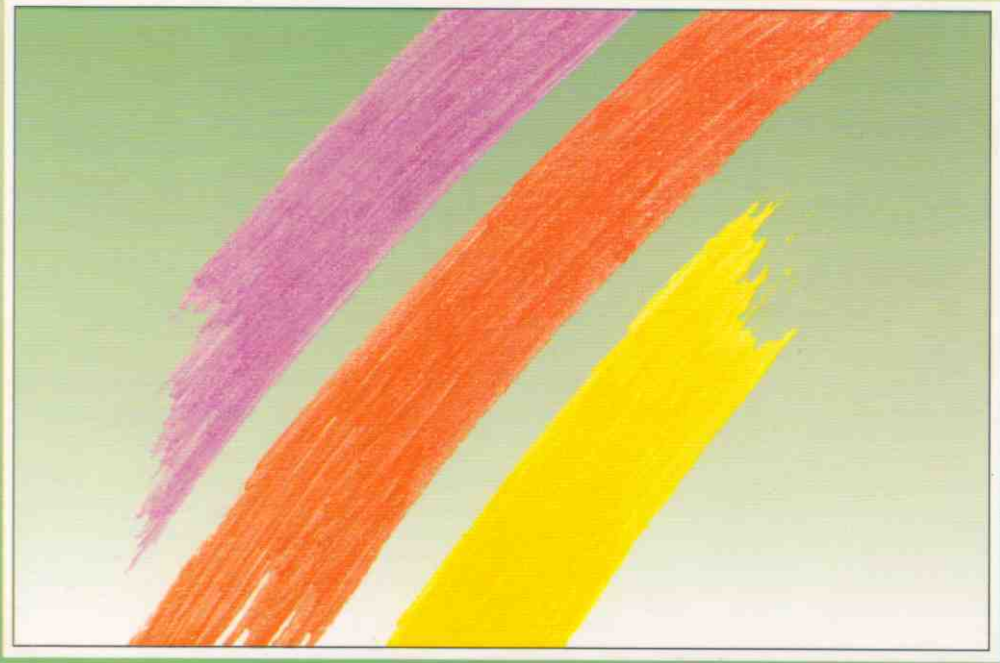
Quantitative Methods in Economics
(Multiple Criteria Decision Making XII)



Proceedings of the International Conference
June 2 – 4, 2004
Virt, Slovak Republic

The Slovak Society for Operations Research
Department of Operations Research and Econometrics
Faculty of Economic Informatics
University of Economics in Bratislava

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Technical Editor: *Martin Lukáčik, University of Economics, Bratislava*
Web Editor: *Martin Lukáčik, University of Economics, Bratislava*
Web: *http://www.fhi.sk/sk/katedry/kove/ssov/papers*

ISBN 80-8078-012-9

Content

APPROXIMATION OF STEPWISE OBJECTIVE FUNCTION IN OPTIMAL CARGO TRAIN ROUTING PROBLEM

PETER BEDNÁR

1. INTRODUCTION

The task to be solved in the optimal cargo train routing problem is creation of routes for wagons transportation. These routes are then represented by trains in the train schedule. So the train routing problem is not concerned with building of physical infrastructure, but with planning of network of trains, which would provide transport of wagons. The task is to determine between which marshalling yards should the trains be dispatched and how should the flows be transported with this trains. The aim of optimization of train routing problem is to find solution that minimizes the transportation costs on the network. As all mathematical models, the mathematical model of the problem describes the real world problem only approximately. In the paper I would like to present one of the inaccuracies of the mathematical model and suggest a way of decreasing its impact on the solution.

2. MATHEMATICAL MODEL OF CARGO TRAIN ROUTING PROBLEM

The optimal cargo train routing problem can be stated as a network design problem. It is a task of selecting a subset of edges as connections, which would serve for handling the transportation demands on the network.

For cargo train routing problem we need data about the possibilities of network creation, which can be given as a graph $G = (V, H)$, where V is a set of marshalling yards between which should the transportation demands be satisfied and H is a set of all edges, which can be used as connections and are potential candidates for selecting in the connections set. The aim of the train routing optimisation is to create a transportation network $G_1 = (V, R)$, where $R \subset H$ is a set of connections, which would serve to handle the transportation demands.

The transportation demands are described as O-D matrix $P = \{p^{rs}\}$, in which for every pair of marshalling yards (r, s) the value of p^{rs} represents the daily amount of wagons to be transported from the yard r to the yard s. For evaluating the quality of the solution we further need fixed costs f_{ij} representing the costs associated with inclusion of the connection (i, j) in the solution independent on the number of wagons, which would be transported with this connection. Next we need costs c_{ij} representing costs of transporting one wagon with connection (i, j). The planned network can be then expressed by matrix $Y = \{y_{ij}\}; (i, j) \in H$, where y_{ij} is bivalent variable taking value 0 if the connection (i, j) is not included and value 1 if connection (i, j) is included in the connection set. Further for every flow p^{rs} we need to model if the flow (r, s) is transported with the connection (i, j). This we model with the bivalent variables x_{ij}^{rs}, taking value of 1 if flow (r, s) is transported with connection (i, j).

The task of the network design problem is to select such set of connections R and such flow routes, which minimizes the value of objective function. The mathematical model of the cargo train routing problem can be written as follows

$$\text{minimize} \sum_{(i,j)\in H} f_{ij} \cdot y_{ij} + \sum_{(r,s)\in K} \sum_{(i,j)\in H} \left(c_{ij} \cdot p^{rs}\right) \cdot x_{ij}^{rs} \tag{1}$$

with constraints

$$x_{ij}^{rs} \le y_{ij} \qquad \text{for} \ (i, j) \in H \tag{2}$$

$$\sum_{(i,k)\in H} x_{ik}^{rs} - \sum_{(k,j)\in H} x_{kj}^{rs} = -1 \qquad \text{for} \ k = r \tag{3}$$

$$\sum_{(i,k)\in H} x_{ik}^{rs} - \sum_{(k,j)\in H} x_{kj}^{rs} = 0 \qquad \text{for} \ k \ne r,\ k \ne s \tag{4}$$

$$\sum_{(i,k)\in H} x_{ik}^{rs} - \sum_{(k,j)\in H} x_{kj}^{rs} = 1 \qquad \text{for} \ k = s \tag{5}$$

$$y_{ij} \in \{0, 1\} \tag{6}$$

$$x_{ij}^{rs} \in \{0, 1\} \tag{7}$$

Constraints (2) provide a relation between variable y_{ij} and flow variables x_{ij}^{rs}. Constraints (3) - (5) represent so-called flow preservation constraints. More details about the model can be found in [2]

3. INPUT COSTS FOR CARGO TRAIN ROUTING PROBLEM

Let's have closer look on the objective function and on the costs, which it is created of.

$$\sum_{(i,j)\in H} f_{ij} \cdot y_{ij} + \sum_{(r,s)\in K} \sum_{(i,j)\in H} \left(c_{ij} \cdot p^{rs}\right) \cdot x_{ij}^{rs} \tag{8}$$

The objective function consist of two terms, where the first term expresses the costs dependent only on the inclusion of connection (i, j) in the solution and second term expresses the costs of wagons transportation with the given set of connections.

f_{ij} – costs of connection independent of the amount of wagons transported with it.

$$f_{ij} = W_{ij} + d_{ij} \tag{9}$$

where $w_{ij} = 12 \cdot M_j \cdot nc$ are waiting costs and M_j is the norm for maximum allowed train length for marshalling yard j, nc are costs for an hour of waiting and d_{ij} are the costs for transportation from yard i to yard j independent on the amount of transported wagons.

c_{ij} – total costs for transporting one wagon from yard i to yard j.

4. STEPWISE OBJECTIVE FUNCTION

One of the inaccuracies of this mathematical model is the cost for transportation from yard i to yard j independent on the amount of transported wagons. The value of d_{ij} represents in reality the cost for transporting a train between yards (i,j) and as such is not really independent on the amount of wagons transported with connection (i,j), but after surpassing the norm M_j of the yard j grows by value d_{ij}. The costs for transportation between yards (i,j) are therefore $dp_{ij} = \left\lceil \frac{q_{ij}}{M_j} \right\rceil \cdot d_{ij}$ where $q_{ij} = \sum_{r,s \in K} p^{rs} \cdot x_{ij}^{rs}$ denotes the amount of wagons transported with connection (i, j).

The objective function is as follows

$$\min \sum_{(i,j)\in H} \left(w_{ij} + \left\lceil \frac{q_{ij}}{M_j} \right\rceil \cdot d_{ij} \right) \cdot y_{ij} + \sum_{(r,s)\in K} \sum_{(i,j)\in H} c_{ij} \cdot p^{rs} \cdot x_{ij}^{rs} \tag{10}$$

Graphical representation of the fixed costs is shown in the figure 1.

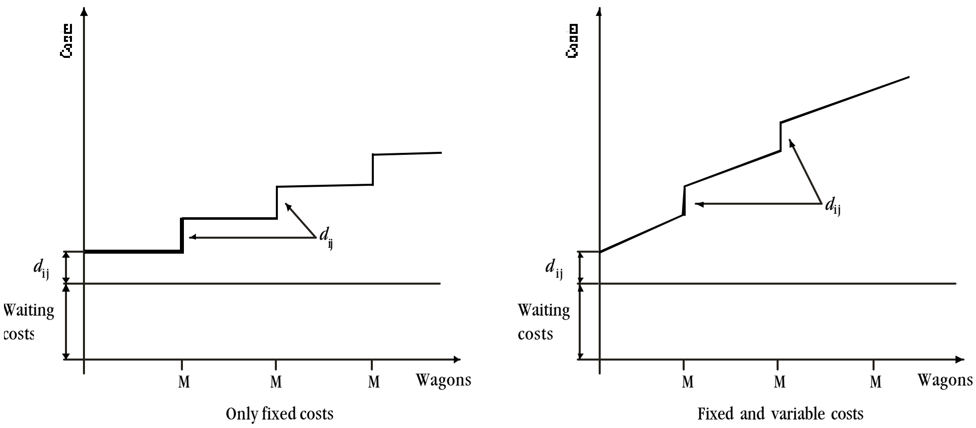

Figure 1

The mathematical model used up to now, written in [2], does not take into account this stepwise increase of fixed costs (see Figure 2).

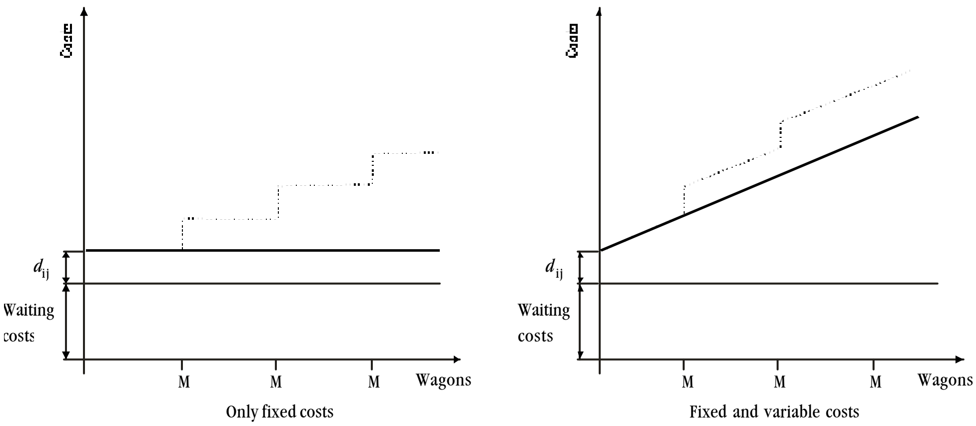

Figure 2

After surpassing the norm M_j and after the need to dispatch another train the cost does not increase. This presents a problem mainly by marshalling yards where the norm is small. The norm for maximum train length actually depends on technological equipment of the marshalling yard and expresses the ability of yard to process trains. Increasing the number of trains can lead to situation when the marshalling would not be able to process all incoming trains. To prevent excessive increase of train amount transported over connections that have small norm it would be appropriate to include a way of increasing the costs when the number of dispatched trains increases.

5. APPROXIMATION OF STEPWISE OBJECTIVE FUNCTION

We would approximate the stepwise increase of the objective function value after surpassing the norm M_j with linear function as shown on the figure 3.

The fixed costs are given by equation (11).

$$f_{ij} = w_{ij} + d_{ij} + \frac{d_{ij}}{M_j} \cdot q_{ij} \tag{11}$$

where $q_{ij} = \sum_{r,s \in K} p^{rs} \cdot x_{ij}^{rs}$. The objective function is then following

$$\min \sum_{(i,j)\in H} \left(w_{ij} + d_{ij} + \sum_{(r,s)\in K} \frac{d_{ij}}{M_j} \cdot p^{rs} \cdot x_{ij}^{rs} \right) \cdot y_{ij} + \sum_{(r,s)\in K} \sum_{(i,j)\in H} c_{ij} \cdot p^{rs} \cdot x_{ij}^{rs} \tag{12}$$

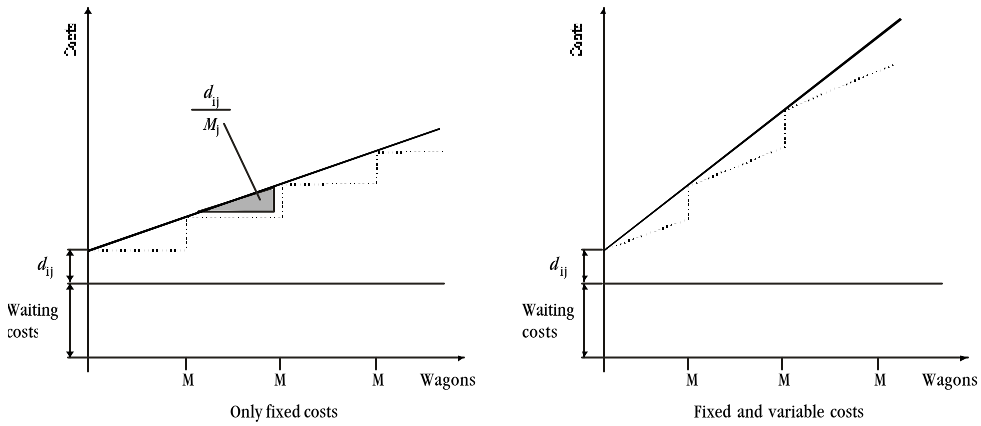

Figure3

However this objective function is no longer linear, but because of constraints (2) from mathematicalmodel we can rearrange the objective function to following form:

$$\min \sum_{(i,j)\in H}\left(w_{ij}+d_{ij}\right)\cdot y_{ij} + \sum_{(r,s)\in K}\sum_{(i,j)\in H} c_{ij}\cdot p^{rs}\cdot x_{ij}^{rs} + \sum_{(r,s)\in K}\frac{d_{ij}}{M_j}\cdot p^{rs}\cdot x_{ij}^{rs} \qquad (13)$$

and further to form

$$\min \sum_{(i,j)\in H}\left(w_{ij}+d_{ij}\right)\cdot y_{ij} + \sum_{(r,s)\in K}\sum_{(i,j)\in H}\left(c_{ij}+\frac{d_{ij}}{M_j}\right)\cdot p^{rs}\cdot x_{ij}^{rs} \qquad (14)$$

It is possible to estimate the maximal error we would make by this approximation compared to stepwise objective function. The error for every connection (i,j) never exceeds the value d_{ij}. So the error caused by approximation would be less or equal to $\sum_{(i,j)\in H} d_{ij}$.

With simple adjustment of approximation function we can obtain approximation with lower maximal error. This adjustment is shown in the figure 4.

The objective function is then given by equation (15).

$$\min \sum_{(i,j)\in H}\left(w_{ij}+d_{ij}+\sum_{(r,s)\in K}\frac{d_{ij}}{M_j}\cdot p^{rs}\cdot x_{ij}^{rs}-\frac{d_{ij}}{2}\right)\cdot y_{ij} + \sum_{(r,s)\in K}\sum_{(i,j)\in H} c_{ij}\cdot p^{rs}\cdot x_{ij}^{rs} \qquad (15)$$

After rearranging we obtain the linear objective function in form

$$\min \sum_{(i,j)\in H}\left(w_{ij}+\frac{d_{ij}}{2}\right)\cdot y_{ij} + \sum_{(r,s)\in K}\sum_{(i,j)\in H}\left(c_{ij}+\frac{d_{ij}}{M_j}\right)\cdot p^{rs}\cdot x_{ij}^{rs} \qquad (16)$$

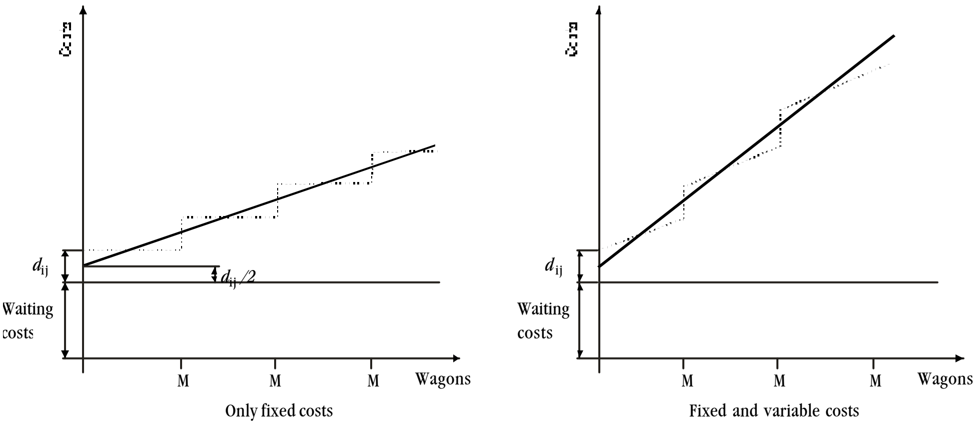

Figure 4

which has error less or equal to $\sum_{(i,j)\in H} \frac{d_{ij}}{2}$

6. EXPERIMENTS

To verify the asset of proposed approximations I have performed following experiments. All experiments were carried out on the data of railway network of Slovak republic. There were 5 variants of input data and for every variant of input data, three different variants of the objective function were used.

- original variant of objective function given by eq. (8)
- approximation variant 1 with objective function given by eq. (14)
- approximation variant 2 with objective function given by eq. (16)

Table 1

Name of experiment	variant of input data	Variant of objective function
ex10a ex10b ex10c	10 marshalling yards, fixed costs are 8% of total costs of train	original variant variant 1 variant 2
ex10e ex10f ex10g	10 marshalling yards, fixed costs are 30% of total costs of train	original variant variant 1 variant 2
ex42a ex42b ex42g	42 marshalling yards, fixed costs are 8% of total costs of train	original variant variant 1 variant 2

Name of experiment	variant of input data	Variant of objective function
ex42c ex42d ex42i	42 marshalling yards, fixed costs are 30% of total costs of train	original variant variant 1 variant 2
ex42e ex42f ex42h	42 marshalling yards, fixed costs are 8% of total costs of train, only domestic flows were included	original variant variant 1 variant 2

The solutions for experiments with 10 marshalling yards are optimal solutions obtained with XPRESS. The solutions of experiments with 42 marshalling yards are suboptimal solutions obtained with heuristical pseudogradient method, which can be found in [Cenek].The results of experiments are in table 2.

Table 2

Name	Costs	Real costs	Error	Number of relations	Average number of trains	Average number of wagons
ex10a	34229.406	36506.421	2277.015	53	10.11	358.7978
ex10b	36655.564	36506.421	-149.143	53	10.11	358.7978
ex10c	36500.031	36511.600	11.569	54	9.926	351.209
ex10e	38610.965	47140.118	8529.154	63	8	273.2719
ex10f	47799.796	46963.397	-836.399	64	7.781	268.3024
ex10g	47004.870	46940.091	-64.779	68	7.265	248.8808
ex42a	45316.278	47766.268	2449.990	131	4.219	146.9504
ex42b	48118.314	47885.288	-233.026	128	4.41	153.2465
ex42g	47785.499	47802.987	17.488	134	4.151	143.8056
ex42c	38805.254	48593.814	9788.560	111	5.101	181.1439
ex42d	49139.956	48287.916	-852.040	120	4.639	164.974
ex42i	47692.451	47724.262	31.811	122	4.506	159.548
ex42e	13657.366	14126.278	468.912	101	1.883	50.0886
ex42f	14245.306	14033.106	-212.200	100	1.912	52.2689
ex42h	14024.392	14028.280	3.888	100	1.912	50.6955

From results arises, that the error of the model compared to stepwise objective function given by eq.(10) is lower in solutions obtained by proposed approximations, where approximation variant 2 is better as variant 1.

From the view point of real costs is in some cases original variant better then proposed approximation, but the difference is not significant. In cases where the fixed costs are 30% of total costs are proposed approximations better than original model. When we compare the average number of trains and average number of transported wagons we can notice the effect of approximations on more even distribution of flows.

7. CONCLUSION

Performed experiments showed that the value of objective function obtained by solving the mathematical model with proposed approximation is closer to real stepwise objective function. Experiments also verified the influence of approximations on decreasing amount of trains dispatched over connections with lower norm.

Although the results obtained with original model when the fixed costs are 8% of total train costs are lower than those obtained in variants with approximations, the difference in objective function value is not significant. I would recommend the use of the proposed approximation of stepwise objective function.

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OPTIMIZATION OF PRODUCTION PROCESSES

IVAN BREZINA, ZUZANA ČIČKOVÁ, MARIAN REIFF

Production logistics supply production process from source to the sink, with movement of intra-firm sources, semi finished products, their warehousing and the optimal distribution to workplaces and organization of production. It is possible to reach efficient organization of production through suitable sequencing and scheduling of production operations in chain. Scheduling theory aims at solving time problem or space allocation of different operations through one or multiple service objects. Scheduling models (in literature often called sequencing tasks) are connected with determination of the procedure how to run the different operations through one or multiple service objects. These models are mainly used to solve production logistics issues.

In the analysis of effective use of service objects with connection to their operations, visual tools, such as Gantt diagram, Gozinto graphs, Sched-U-diagrams, and flux scheme of production are used. In production logistics methods based on graph theory, integer programming, models of enterprise (input-output), queueing models and dynamic programming are often used ([6], [7]). Except these classical approaches a huge number of heuristics was well developed, on which we will focus.

The core of simple scheduling models is sequencing and scheduling of operations on one working object (machine). Among more complicated models (based on number of service objects) belong operations sequencing and scheduling on more than two service objects. Scheduling models of flux-organized systems are also complicated models. Their core is scheduling of fixed sequence operations to each object (there are elaborated simple heuristics for two, three or more objects). In case sequencing and scheduling operations according to the type or kinship is required, we are talking about sequencing models of phase systems.

In case of multiple service objects (we assume, they are in serial order, and so form a line), when the order of processing of each operation is important (equal order of realization of each operation), we are talking about FLOW SHOP systems. When service objects are in serial order and it is possible to realize an arbitrary order crossing of each operation (within complex tasks that order is usually equal), we are talking about JOB SHOP systems. If there is given an aggregate of operations without categorization to joint tasks (the operations don't have a given order in the task) and aggregate of service objects, (each operation is assigned to one service object and the order of realization of operations is arbitrary) we are talking about OPEN SHOP systems ([8]).

A special kind of decision-making models are models of assembly line balance, which are characteristic for mass assembly line production. These models are very extensive in practical applications.

In each task of scheduling n operations to m service objects (machines), it is possible to specify these input data (quantities):

- the set of operations n and their order $\{o_1, o_2, \ldots, o_n\}$,
- the set of service objects m and their order $\{s_1, s_2, \ldots, s_m\}$,
- the due date required to complete operation d_j;
- the processing time of j-th operation on i-th service object t_{ij}.

For evaluation of scheduling or sequencing are used relatively simple optimization criteria. One group of criteria consists of the maximum of completion time of each operation and the other consists of the sum of completion time of these operations. The goal of these both groups of criteria is the effort to minimize the cost connected to the time of realization of operations, so with the time interval spent in the production system, respectively the effort to minimize the fall behind schedule (lateness) compared to the required due date of finishing operation, and the penalty following from it, or effort to minimize the number of late tasks.

According to the number of service objects it is possible to divide algorithms to system with one or multiple service objects. We are talking about a single service object system, if we have one available service object (machine) to serve n operations. To solve them we often use one of the simple heuristics for single service object:

- Moore's algorithm,
- Smith's algorithm,
- Lawler's algorithm.

Moore's algorithm (Moore 1968) comes out from the requirement, that no operation is executed late. In case it is not possible to accomplish this requirement, the next criterion can be the minimization of the total number of late operations. Moore's algorithm allows to reach this goal, and so to gain such order of operations on the service object, in order to minimize the number of late operations (lateness).

Smith's algorithm is based on a different approach. The goal is to make such an operations schedule, that no operation is overdue according to desired term (due date) of completing operation d_j.

The realization of *Lawler's algorithm* (Lawler 1973) secures to minimize the total sum of weighed lateness (overdue). In scheduling of production process operations, there are two conditions directly influencing sequencing of operations of production process:

- precedence constrains, which restrict operations, the realization of which is constraint to follow the processing (technological or technical connection),

- weighting factors w_j, which represent the importance of each operation (job) - the preference of each operation is given as follows: the lower the weight, the higher the preference of relevant operation.

For the solution of FLOW SHOP systems (scheduling system of operations on serial ordered service objects with equal order of operations) except other approaches (like Branch and bound method used to solve integer programming tasks) different heuristics are used and the best-known are:

- Johnson's algorithms,
- Palmer's heuristic,
- Grupt's heuristic,
- Campbell's, Dudek's and Smith's heuristic.

The best-known *Johnson's algorithm* (Johnson 1954) is designated for two following service objects M_1 and M_2. The target is to minimize the maximum values of processing time of operations, or minimize the maximum over all time to complete all the operations, which is known as the makespan of the schedule. In case we have got three service objects (machines) M_1, M_2, M_3, and each operation must be processed first on M1, then on M2, and finally on M3, we can use *Johnson's algorithm* for three-object FLOW SHOP.

The solution of the given problems seems to be easy. But we should realize, that with the growing number of service objects also the number of solution variants of each task grows. These tasks belong to combinatorial optimization tasks. Realization of *n* operations gives us *n!* possible combinations of scheduling operations on single service object. If we consider *m* service objects, the number of possible combinations is $(n!)^m$. To find an effective schedule of realization of operations means to optimize the effectiveness (purpose) function, in our case to minimize the overdue (downtime) of each service object. For solving such tasks the complete enumeration method or other heuristics are used.

With *Palmer's heuristic* it is possible to construct a permutation schedule, in which the order is equal for each *n* operations on all *m* service objects (the number of service objects is not limited with three objects like in the previous algorithm). The goal is again to optimize the overdue (downtime) of each service object, thus minimize the maximum values of processing time of operations, or minimize the maximum over all time to complete all the operations, consequently we have got selected solution from $(n)^{m-1}$ possibilities.

With *Grupt's heuristic* it is possible to make such a permutation schedule, the target of which is also to minimize the overdue (downtime) of each *m* service object, thus minimize the maximum values of processing time of *n* operations, or minimize the maximum over all time to complete all the operations.

Campbell's, Dudek's and Smith's heuristic (Campbell 1970) has got *m-1* stages, and in each stage from the original multiple-object problem a two-object problem results. For the two-object problem we can compute suboptimal permutation of tasks, with Johnson's algorithm for two following service objects. After $m-1$ stages we get $m-1$ permutation schedules and consecutively select one with the minimum overdue (down time).

For solving the JOB SHOP systems (systems of scheduling operations on serial ordered objects with equal order of operations in one given task and realization of each task is in a random order) for example graph method, integer programming and heuristics are used. The best-known heuristics are:

- Johnson's algorithm for two non-following service objects,
- algorithm generating active schedules (Giffler and Thompson),
- algorithm for generating nondelay schedules,
- Bottleneck heuristic.

In case, that *m* service objects are in parallel order and n operations should be realized on them and each operation can be realized on any random service object at processing time t_{ij}, we can talk about parallel identical service objects (facilities).

Relatively well-known algorithms used for scheduling operations on parallel ordered service objects are among others:

- McNaughton's algorithm,
- Hu's algorithm,
- Muntz-Coffman algorithm (Muntz 1969).

McNaughton's algorithm (McNaughton 1961) comes from the assumption, that we have *m* parallel ordered service objects and must realize *n* operations on them. These operations can be interrupted and one operation can't be processed on two service objects at the same time.

Hu's algorithm (Hu 1961) is designated to make up working schedule for parallel objects. In this case processing time of operation o_j is not considered (each operation has a simple processing time $t_j=1$) and precedent realization on the set of operations has a shape of a production tree. This means each node has not more than one follower. The relations between the nodes are given by technical and technological connections. The task is to match *n* operations to *m* service objects.

In bigger production plants we often see that in the production process the product goes through several working places and the production process is realized in regular intervals. It is typical for assembly line production and assembly line in various industrial sectors, mostly in mechanical engineering and high-tech industries.

In assembly line production, it is not possible to interrupt technological connection of jobs and it is economical to pay attention to task each work place equally. In literature this problem is known as assembly line balancing. The core of the problem is to assign single operations to workplaces with no constraints in technical and technological or assembly connection to realized job operations and well-proportioned distribution of work to each work place. If the assembly line is not balanced, down time will occur at workplaces.

Typical problems are:

- minimizing the number of workplaces in given time, during which the product stays on each workplace (production tact)
- minimizing the time, during which product stays on each workplace, at a given number of workplaces,
- minimizing the unutilized time of whole assembly line, if it is possible to choose the time, during which the product stays on each workplace at given number of workplaces,

Generally it's possible to divide the methods for solving the given problems into:

- *analytical (exact) methods*, which give optimal solution, but for most difficult computation they are not suitable for solving practical, usually very complex and extensive issues.
- *heuristics*, are based on empirical observation, which may not lead to optimal so-lution, but guarantee a relatively simple and economical way to find an effective solution.

Best-known analytical approaches are:

- approaches of integer programming (Bowman, White),
- methods of dynamic programming (Held, Karp, Shareshian, Jackson)
- combinatorial methods – methods based on precedent matrix or Branch and bo-und method (Elhambry),
- methods of network modeling - based on finding the shortest way (Gutjahr, Ne-mhauser, Mansoor),
- simulation methods

One of the best known and most applied heuristic is the weight technique (Mansoor). This method comes out from construction of orientated network (prog-ress diagram), which describes technological, technical or assembly connection of single operations.

The core of the approach of this method is in assigning the particular quantity (weight w_j) to each single operation. Computed weights w_j evaluate the meaning of each operation and according to them operations are categorized to workplaces. The weights w_j are computed as a sum of processing time of the given operation and of the weights of all following operations, which directly depend on that operation. This method gives to those operations preferential treatment on which the following operations depend.

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THE POSSIBILITIES OF FURTHER DEVELOPMENT OF ALGORITHMS FOR SOLVING THE UNCAPACITATED FACILITY LOCATION PROBLEM

ĽUBOŠ BUZNA

1. INTRODUCTION

The uncapacitated facility location problem is a broadly exploited in the area of optimization of private and public services, where is possible to model whole set of location problems using this problem as a solving pattern (Janácek, 2004). It appears in literature as a uncapacitated facility location problem, or also simple plant location problem.

The goal is to chose, so called, facilities, what could be stores, supermarkets, offices, hospitals, etc., from finite set *I* (candidates for placing a facility) and to assign to them customers defined with set *J*, what could be dwellers of autonomy areas, buyers, or smaller shops, etc. There is a requirement to assign every customer, just to one facility. The corresponding problem can be written in the form:

$$\text{Minimize} \quad \sum_{i\in I} f_I \cdot y_I + \sum_{i\in I}\sum_{j\in J} c_{ij} \cdot z_{ij} \tag{1}$$

Subject to

$$\sum_{i\in I} z_{ij} = 1 \qquad \text{for } j\in J \tag{2}$$

$$z_{ij} \le y_i \qquad \text{for } i\in I \text{ and } j\in J \tag{3}$$

$$y_i, x_{ij} \in \{0,1\} \qquad \text{for } i\in I \text{ and } j\in J \tag{4}$$

where y_i are binary variables obtaining a value *1*, if the facility is located in a candidate place *i* and the value *0* otherwise. The variables z_{ij} are also binary variables with value *1* in a case of assigning the customer *j* to the facility *i* and *0* otherwise.

To solve this problem was proposed big amount of algorithms. According the present literature (Crainic, 2002), a procedure *DualLoc* (Erlenkotter, 1978) is still appearing one of the most effective. At the groundwork of this procedure were created algorithms *BBDual* (Janácek, 1998) and *PDLoc* (Körkel, 1989), which employ a branch and bound method for seeking out the optimal solution. The paper is organized as follows. We are briefly introducing the principle of *DualLoc* procedure and describing the differences between *PDLoc* and *BBDual* in sections 2, 3 and 4. Section 5 compares the efficiency of *BBDual* and *PDLoc*, to solve the same sized "standard" location problems and the maximum distance problem. To conclude the paper the section 6 sums up the possibilities for further development of algorithms *BBDual* and *PDLoc*.

2. PROCEDURE DUALLOC

Procedure *DualLoc* allows to find a solution of problem (*1*) – (*4*) with efficiently way. The process of solving is based on finding the feasible solution, as good as possible (the value of objective function is as high as possible) of the problem (*5*) - (*6*), what is dual problem to the LP relaxation of problem (*1*) - (*4*).

$$\text{Maximize} \qquad z_D = \sum_{j \in J} \nu_j \tag{5}$$

Subject to

$$s_i = f_i - \sum_{i \in I} \max\left(0, \nu_j - c_{ij}\right) \geq 0 \qquad \text{for } j \in J \tag{6}$$

To find this solution was proposed a method, called dual ascent. It is increasing in a cycle the values of variables z_D, until all variables are blocked by the constraints (*6*). The value of objective function (*5*) is the lower bound for optimal solution of original problem (1) - (4). Feasible primal integer solution is then derived from dual solution, with the intention, to satisfy the complementary conditions (7) - (9), as close as possible. The value of objective function z_P, of this primal solution, is the upper bound of the actual branch.

$$\left[c_{ij} - \nu_j + \max\left(0, \nu_j - c_{ij}\right)\right] \cdot x_{ij} = 0 \qquad \text{for } j \in J,\ i \in I \tag{7}$$

$$s_i y_i = 0 \qquad \text{for } i \in I \tag{8}$$

$$\left(y_i - x_{ij}\right) \max\left(0, \nu_j - c_{ij}\right) = 0 \qquad \text{for } j \in J,\ i \in I \tag{9}$$

If there is a gap between lower and upper bound, the obtained dual solution is further modified with a procedure called dual adjustment, which allows more, to increase the value of objective function. The dual adjustment procedure is performing the dual alternations (is trying in cycle to identify a variable z_D, which decreasing about a positive value, allows to increase more then one others variables about the same value) and is subsequently updating the primal solution.

The optimal solution we can find using a branch and bound scheme. The branching process run over the variables y_i. As a lower bound is used the value of objective function of relevant dual subproblem. The upper bound is the value of objective function of primal problem generated from dual subproblem.

3. ALGORITHM PDLOC

The algorithm *PDLoc* (Körkel, 1989) was derived from the procedure *DualLoc*. The optimal solution is traced with the branch and bound approach, using a best bound strategy. The disadvantage of this strategy is the inauspicious computer memory consumption caused by the necessity, to store all bright dual solution, for the occasion of their future using. Algorithm employs more effectively working procedure dual ascent and also more sophisticated procedure dual adjustment, where is the number of iteration determined according the size of problem (number of facilities and customers). The demerit place of procedure dual ascent, is a case, when is a big difference between coefficients f_i and c_{ij} ($f_i >> c_{ij}$). Then is the dual ascent very time consuming, because it takes a long time until it is filled up the "capacity", for variables z_D escalation, restricted by the constraints (*6*). This difficulty of algorithm *PDLoc*, eliminates the incorporating of multi dual ascent procedure. This procedure at first sets the variables z_D at the highest value as classical dual ascent, it can also break the constraint (*6*), after that the variables z_D are modified to fulfil constrains (*6*) and then is used the classical dual ascent procedure.

The algorithm *PdLoc* has also improved procedure for finding primary solution. It implements a changes in a order of opening facilities always, when procedure is called. The best bound strategy opens up the possibility for fixating of variables y_i, to values *1* or *0* during solving process, what reduces the size of problem. To verify, if we can fix some variable to values *1* or *0,* or not, is considerably time consuming. In order to save a computation time, we fix variables only, when it is possible to fix a greatly volume of variables y_i concurrently.

The algorithm is storing in memory, only dual solutions, by reason of, we must after taking out the solution from memory, to recalculate the primary solution. We need it to do, to find a variable for branching. To quickly recalculate the primary solution, was proposed a special heuristics approach. This heuristics do not give such good solution as previous one, but this solution is more proper for choosing the branching variable. As the branching variable is used a variable, which breaks the complementary constraints (*6*) most considerably.

To keep a length of paper in reasonable limits, we have only sketched the main attributes of algorithm *PDLoc*, to have a more detailed description of this algorithm, we can suggest to the reader the paper (Körkel, 1989).

4. ALGORITHM BBDUAL

The foundation of formulation algorithm *BBDual* (Janácek, 1998), was also procedure *DualLoc*. To trace optimal solution, the branch and bound method with depth first strategy is employed. This strategy is more memory savings and it requires less data movement operations. The disadvantage is, that it is not so simple to reduce size of problem during the solving process.

In the (Buzna, 2003) are described in detail both algorithms and there is present a comprehensive analysis of efficiency of most important procedures separately and also both algorithms in complex. For majority of benchmark the algorithms achieved relatively equivalent time consumption. The algorithm *BBDual* was slightly more efficient, but at a special group of benchmarks, where $f_i >> c_j$, was algorithm *PDLoc* significantly more efficient, as you can see in table tab. 1.

Tab. 1

Size of problem (|I| × |J|)	**PDLoc**		**BBDual**	
	t_{S1} [s]	t_{S2} [s]	t_{S1} [s]	t_{S2} [s]
100 ×2906	0.545	1.414	35.348	0.266
400 ×2906	0.879	3.048	879.516	1.856
700 ×2906	1.295	11.681	1936.640	11.309
1000 ×2906	1.756	15.285	3952.100	20.429

Overall average times in seconds, at 10 benchmarks problems, for algorithms PDLoc and BBDual,

t_{S1} – the times for group of benchmarks where $f_i >> c_{ij}$ (only one facility is placed in optimal solution),

t_{S2} – the times for benchmarks, which have $|I|/2$ placed facilities in optimal solution.

After detailed analysis of computation characteristics both algorithms and reciprocal exchange of positive improvements between both algorithms, we suggested in (Buzna, 2003) our own modifications, what brought a savings in computation time as you can see in table tab. 2.

Tab. 2

Size of problem (|I| × |J|)	**PDLocf**		**BBDualh**	
	t_{S1} [s]	t_{S2} [s]	t_{S1} [s]	t_{S2} [s]
100 ×2906	0.583	0.834	0.121	0.194
400 ×2906	2.384	1.616	1.514	1.109
700 ×2906	7.152	7.810	5.556	5.585
1000 ×2906	14.036	6.650	11.466	7.764

Overall average times in seconds at 10 benchmarks problems for improved version of algorithms PDLocf and BBDualh,

t_{S1} – the times for group of benchmarks where $f_i >> c_{ij}$ (only one facility is placed in optimal solution),

t_{S2} – the times for benchmarks, which have $|I|/2$ placed facilities in optimal solution.

The most markedly have contributed to the decreasing of computation time, the take over of multi dual ascent to the *BBDual* and new way of ordering customers in dual accent procedure.

5. THE MAXIMUM DISTANCE PROBLEM

As we mentioned in section 1, the uncapacitated facility location problem is appropriate to solve the whole set of location problems. One of this problems is the maximum distance problem. In this problem is the customers demand considered as a satisfied, when the nearest facility is placed nearer then distance *D*. The model of maximum distance problem, can be written in following way:

$$\text{Minimize} \sum_{i \in I} y_i \qquad (1)$$

Subject to

$$\sum_{i \in N_j} y_i \geq 1 \qquad \text{for } j \in J. \qquad (2)$$

where y_i are the binary variables, set at value *1*, if there is a placed facility in candidate place *i* and at value *0* otherwise, $N_j = \{i \in I: d_{ij} \leq D\}$. In our experiments we used the benchmarks from testing algorithms *BBDual* and *PDLoc*, which were derived from Slovak road network (Buzna, 2003) and we set $D = 80$ km. To solve the maximum distance problem with algorithms *BBDual* and *PDLoc* we transformed the uncapacitated facility location problem by setting $f_i = 2$ for $i \in I$, and $c_{ij} = 1$ for $j \in J$ and $i \in N_j$, and $c_{ij} = 4$ otherwise.

Tab. 3.

Size of problem	PDLocf [min]	BBDualh [min]
100 × 2906	7,99	0,23
200 × 2906	533,13[1]	> 24 hours[2]
300 × 2906	> 24 hours	> 24 hours

Overall average times, at 10 benchmarks problems in minutes, for improved version of algorithms PDLocf and BBDualh solving the maximum distance problem.

As it is shown in table tab. 3, both examined algorithms achieved significantly worse computational times at the benchmark problems of the same size, as in table tab. 2, where were coefficients c_{ij} set proportional to the distances between object in

1 In time limit 24 hours were solved only 7 from 10 benchmarks.

2 In time limit 24 hours was solved only one benchmark problem, in time 780,42 minutes.

road network. This settings is more favourable for algorithms, because difference between coefficients c_{ij} and their distribution, predetermined with spatial spreading of customers, enables quickest elimination of gap between lower and upper bound. The similar deterioration of computation properties of exact algorithms for solving uncapacitated facility location problem was observed in problems with randomly generated coefficient c_{ij} and f_i. To limit this adverse phenomenon, we tried to use this settings of coefficients, $f_i = 2$ for $i \in I$, and $c_{ij} = 1$ for $j \in J$ and $i \in N_j$, and $c_{ij} = 4 + \underline{c}_{ij}$ otherwise. The coefficients $\underline{c}_{ij}$ were taken over from original location problems. This attempt did not bring the remarkable achievement.

6. THE POSSIBILITIES OF FURTHER DEVELOPMENT OF ALGORITHMS BBDUAL AND PDLOC

As was signified in previous parts of article, there is still the motivations to dedicate a effort, to improve the computational properties of both exacts algorithm *BBDual* and *PDLoc*. The reasons are in size of uncapacitated facility location problems coming from praxis and also in adverse behaviour of algorithms in special cases, like the maximum distance problem.

The important attribute of both algorithms is a finding the trade offs between, how much time to spend to improve a dual solution in procedure dual ascent and dual adjustment and how many branches to searching, depending at the size of the problem. Here is a huge area to experiment with algorithms settings during dual adjustment, where would be possible to intensify more thorough the number of performed operations depending on size of problem. There would be also possible, to estimate the difficulty in accordance with distribution of coefficients in objective function and to set the attributes of algorithms.

In front of the process of branching in algorithm *BBDual*, is possible the fixation of variables y_i to incorporate, to reduce the size of problem. There is conceivable try to define more effective criteria for selection of variable for branching as well. The separate section of future research, could be a mentioned adverse behaviour of algorithms *BBDual* and *PDLoc*, at the problems with unnatural settings of coefficients in objective function.

7. CONCLUSIONS

We have briefly introduce the principles of procedure *DualLoc* and have concisely described the derived exact algorithms *BBDual* and *PDLoc*. We have shown their computational properties at practical uncapacitated location problems and have compared them with the maximum distance problem. The maximum distance problem and problems, with unnatural settings of coefficients in objective function, have had very adverse computational time consumption. This phenomenon and also effort to increase a computational efficiency of exacts algorithms for solving uncapacitated facility location problem, is legitimate for praxis and could be topic of a further research.

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Acknowledgement: This work has been supported by research project VEGA 1/0498/03.

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MODEL FOR ANALYSIS AND PROGNOSIS OF DAILY REVENUES OF THE STATE TREASURY OF THE SLOVAK REPUBLIC

MICHAL FENDEK

Abstract In the paper is described the model surroundings for analysis and prognosis of daily revenues of the State Treasury of the Slovak Republic. This model surroundings is aimed at support of efficient operation of the State Treasury System through qualified information about day-to-day short-term forecast of the tax revenues development in the view of the single quarters and respective months of the current year.

Keywords: tax revenues, personal income taxes, corporate income taxes, witholding income taxes, value added tax, excise taxes, state treasury

1. INTRODUCTION

In paper we depict the methodological approach applied with the analysis and prognosis of the State Treasury day-to-day incomes stemming from personal income taxes, corporate income taxes, witholding income taxes, value added tax and excise taxes. The paper presents results for the analysis and forecasting of the State Treasury incomes consisting of

- Personal income taxes, corporate income taxes and witholding income taxes
- Value added tax
- Excise taxes

The aim is to design a prognosis of daily changes on the accounts of the above mentioned tax items while taking into consideration a historical profile of their development, number of non-business days (holidays) in the forecasted month with a possibility for exogenous variant definitions of monthly prognosis of the balances on the mentioned accounts.

The topical knowledge of econometric modelling and software products applicable for economic-mathematical modelling were used for solving of the problem. However, the final objective was to create such a model surroundings that enables the State Treasury employees to make an efficient and autonomous use of the system without requirements for specific expertise in econometrics and economic-mathematical modelling.

2. DATABASE OF THE MODEL AND METHODOLOGY OF ANALYSIS

The initial database for a solving of the project are historical data of State Treasury revenues in the structure provided by Tax Directorate of the Slovak Republic and Customs Directorate of the Slovak Republic. We dispose of complete day-to-day time series concerning daily development of changes in Personal income taxes, corporate income taxes, withholding income taxes, Value added tax and Excise taxes for the period from 01.01.2000 to 30.06.2003 in accordance with the Economic classification of incomes, grants, loans and other financial advances [2] in the following structure:

111001	Tax on dependent activity and emoluments of officers
111002	Tax on business activity and other independent gainful activity
112001	Corporate income tax (residents)
112002	Corporate income tax (non-residents)
113001	Withholding income tax (from private individuals)
113002	Withholding income tax (from legal entities)

Value added tax realised through Tax offices of SR

131001	VAT
131002	Claimed over-deduction of VAT

Value added tax realised through Customs authorities of SR

dph0705	VAT

Excise tax realised through Tax offices of SR

132001	Tax on mineral oils
132002	Tax rebate of paid tax on mineral oils
132003	Tax on alcohol
132004	Tax rebate of paid tax on alcohol
132005	Tax on beer
132006	Tax rebate of paid tax on beer
132007	Tax on wine
132008	Tax rebate of paid tax on wine
132009	Tax on tobacco and tobacco products
132007	Tax rebate on tobacco and tobacco products

Excise tax realised through Customs authorities of SR

spd1791	Tax on mineral oils (inland)
spd2700	Tax on mineral oils (import)
spd2719	Tax on alcohol
spd2727	Tax on wine
spd2735	Tax on beer
spd2743	Tax on tobacco and tobacco products

The methodology applied for a formation of the model allows the State Treasury analysts to solve the following tasks:

(a) to make access to historical time series of daily changes on the accounts of individual tax items in a transparent form,

(b) to make access to historical time series of accumulative daily values of the accounts of single tax items in a transparent form,

(c) based on historical data to generate qualified predictions of development of accumulative daily values of balances on the single tax item accounts with an application of econometric approaches.

(d) to transformate the prognosis of accumulative daily values of balances on the accounts of single tax items in such a way, that it takes into consideration a calendar of a current month in terms of non-working days, i.e. holidays, Saturdays and Sundays,

(e) based on a transformed forecast of development of accumulative daily values of balances on the accounts of single tax items to derive an adequate prognosis of daily changes of balances on the accounts so that, it reflects a calendar of a current month in respect of non-working days,

(f) based on a prediction of profiles of daily changes of balances on the accounts of Personal income taxes, corporate income taxes and witholding income taxes to generate short-term forecasts of daily changes of balances on the accounts under alternatively defined expected balances on the accounts at the end of predicted months,

(g) to generate analogously short-term forecasts of accumulative values of daily balances on the accounts under alternatively defined expected balances on the accounts at the end of forecasted months.

Accumulative daily development of personal income taxes, corporate income taxes and withholding taxes, value added tax and excise taxes is predicted through individual regress equations. Their parameters were estimated in the programme system *Soritec for Windows* via application of *Cochrane-Orcutt* technique used for elimination of stated auto-correlation.

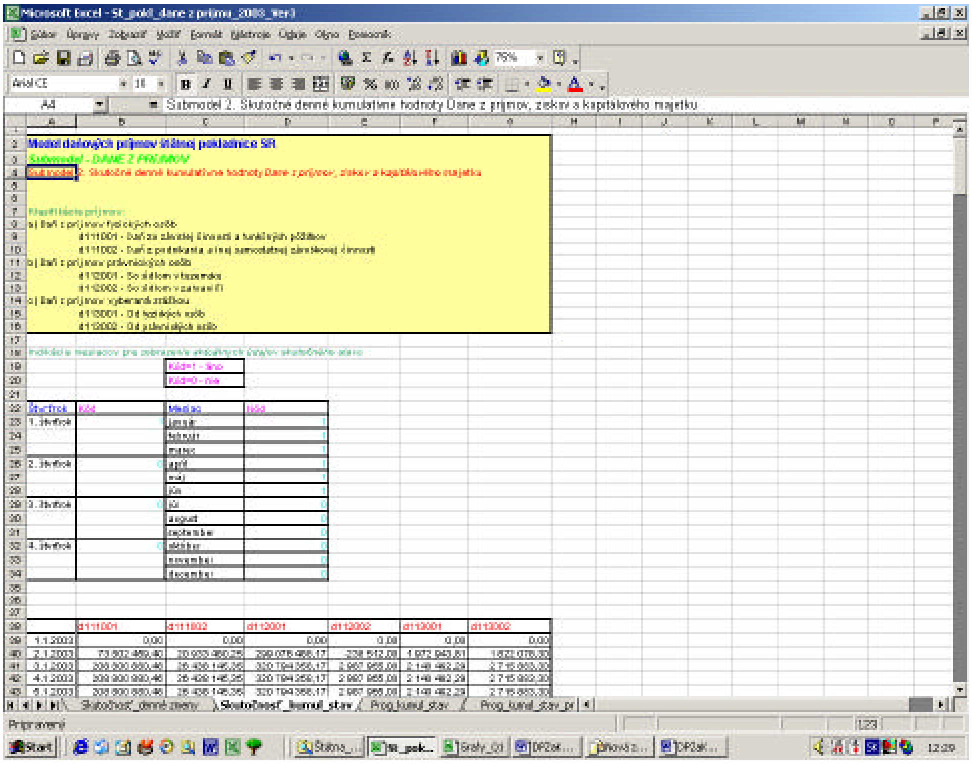

Figure 1

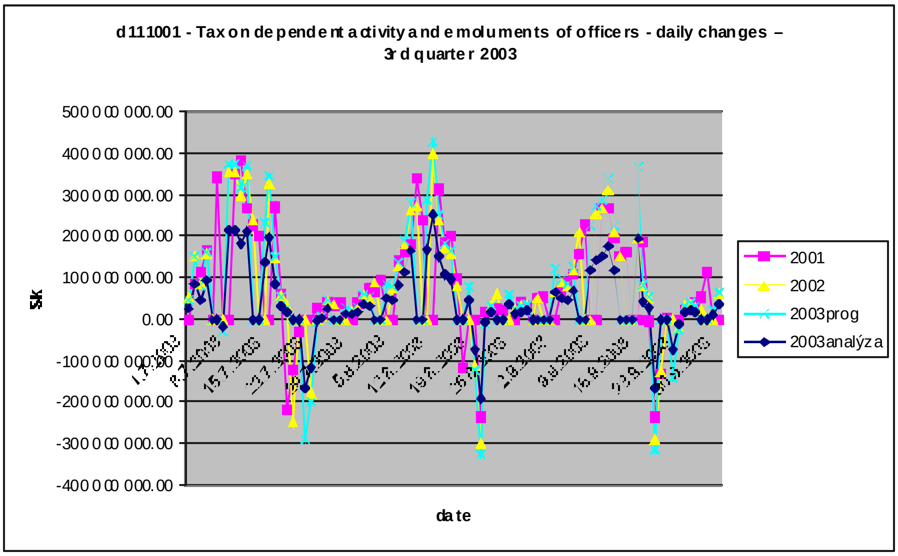

Figure 2

This methodology is realised on the base of Excel model in surroundings of programme system Micosoft ® Excel 2000 and the outputs are generously documented and illustrated in the surroundings of a large graphic support.

The initial display of the real balances on the single accounts of tax incomes, (expected values) set by the user is in Figure 1. The graphical interpretations of the prognosis of personal income tax (tax on dependent activity and emoluments of officers) – daily changes – 3rd quarter is in Figure 2.

3. CONCLUSION

In the paper we presented a model supportive apparatus for analysis and prognosis of the State Treasury revenues stemming from Personal income taxes, corporate income taxes, witholding income taxes, Value added tax and Excise taxes. The historical database of the system of models is formed by daily time series characterizing individual tax incomes of the stated structure of public funds for the period of the years 2000 to 2003Q2.

The outcome of realisation of the model is a daily forecast of revenues of the State Treasury SR for individual months of an observed year, whereby in the course of the year the model enables to update information about finished months. The presented model provides information about prognosis in two regimes of work:

(1) as a result of the prediction following from historical databases of models on the basis of econometric models concerning tax revenues development of the State Treasury of the Slovak Republic,

(2) as a result of the forecast taking into account individual definitions of expected monthly revenues of single tax system items, responding to the anticipated short-term current development of the system

Model solutions of all variants of forecasts of development of individual tax system items are supported by files of illustrating graphs.

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EVALUATION OF UNIVERSITY GRADUATES TECHNICAL EFFICIENCY USING DATA ENVELOPMENT ANALYSIS COMBINED WITH MULTIPLE CORRESPONDENCE ANALYSIS: THE CASE OF THE UNIVERSITY OF FLORENCE

GUIDO FERRARI, TIZIANA LAURETI

Abstract: The aim of this paper is to use a modified two-stage technique to insert the students' characteristics into a technical efficiency evaluation of the Italian university system based on Data Envelopment Analysis (DEA). In the first stage, Multiple Correspondence Analysis (MCA) is applied to a set of variables, which describe the university educational process, to obtain a reduced number of factors that indicate the students' characteristics as well as the faculties' ones. In the second stage, DEA is used to evaluate graduates' technical efficiency based on this reduced number of factors. Evidence from 1998 graduates in the University of Florence is provided to show the potentiality of this procedure to identify the share of efficiency due to students' capacity and that due to Faculties' efficiency in resources supplying.

Keywords: Multiple Correspondence Analysis, Data Envelopment Analysis, Frontier Technical Efficiency.

1. INTRODUCTION

A special interest is growing in the Italian university system on issues related to the measurement of productive efficiency of the universities, and on the construction of related performance indicators. Since a university can be thought of as a multi-product firm (Johnes, 1993), performance indicators might be constructed for different features (e.g. internal performance indicators related to completion spells, exam scores and final degree score, etc.).

By and large, university efficiency analysis has followed three approaches:

(i) a university level approach, where the unit of observation is the institute of higher education itself (Johnes, 1996; Breu and Raab, 1994; Sarrico *et al.* 1997);

(ii) a subject level one, where the unit of observation is a department (Beasley, 1990, 1995; Johnes and Johnes, 1993); and

(iii) an individual level approach, where the unit of observation is the student (Ferrari and Laureti, 2002).

Different methods have also been used, the main distinction being between a parametric approach and a non-parametric one. Parametric techniques that usually have been used include regression methods and limited dependent variable models (see, as an example, Smith and Naylor, 2001). Non-parametric methods have largely been based upon Data Envelopment Analysis (DEA) (Lovell and Schmidt, 1988).

The purpose of this paper is to evaluate the frontier efficiency of the human capital formation in the University of Florence. In particular, we will use DEA to obtain measures of the efficiency in the production of the graduates at the University of Florence in 1998.

Multiple Correspondence Analysis (MCA) is used in this paper to insert students' characteristics into a DEA-based efficiency evaluation procedure. This suggested two-stage procedure allows to directly evaluate the impact of students' characteristics on their performance, by avoiding the bias due to the use of the two-stage procedure that typically is utilized to incorporate environmental effects in DEA[1] .

Furthermore, the use of MCA enables to reduce the number of inputs with minimum loss of information. As the graduates are nested into degree course programmes, our two-stage procedure - that uses the MCA loadings to measure the technical efficiency of the graduates – is inspired by the one suggested by Charnes, *et al.* (1981) and recently developed by Portela and Thanassoulis (2002).

The paper is organised as follows. In the next Section, the human capital formation process in the Italian university is regarded as a "classic" production process. In Section 3, the MCA-DEA based two-step efficiency evaluation procedure is discussed. In Section 4, data from Florence University are discussed and some empirical results from the efficiency estimation are presented. Section 5 concludes with some comments on procedure and results.

2. THE UNIVERSITY PRODUCTION PROCESS OF A GRADUATE

The formation of a graduate in the university can be viewed as a production process, even though characterized by specific features that make it peculiar in the classic production process framework (Ferrari and Laureti, 2002). The university,

1 Indeed, the typical two-stage approach follows a first stage DEA exercise based on inputs and outputs, and a second stage regression analysis aiming at explaining variations in first stage efficiency scores in terms of a vector of observable exogenous variables (e.g. Pitt and Lee, 1981).

through the training, transforms a cultural "raw" material represented by the students coming from high school, into a cultural "refined" material (output), through the utilization of a number of material and non-material inputs, such as teachers, textbooks, class-rooms, computers, libraries, etc. In this context, also the student who is submitted to the educational process is viewed as an input. But he takes more or less actively part in the carrying out of the process and affects the results. Indeed, in the production process, the student plays both active and passive roles. Thus, we look at the student as the production unit who, by using the above inputs supplied by the faculty he/she attends along with other individual inputs (e.g. his/her individual psychological, social, and household characteristics) produces him/herself as a graduate, i.e., as an output possessing a number of characteristics (Figure1).

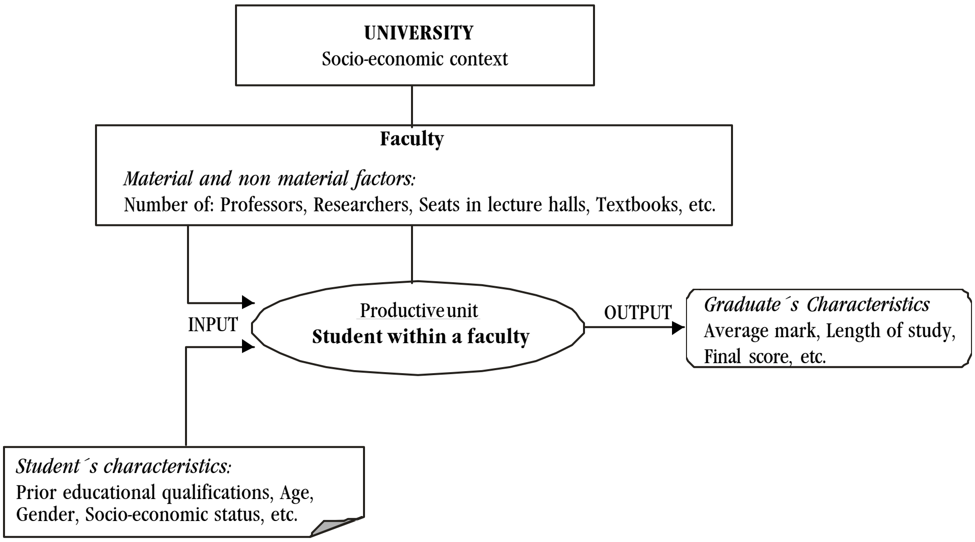


Figure 1: The University Production Process of a Graduate

Hence, we model the production process as an input-output space where the observed points are represented by students within a given faculty. We will perform the analysis of this complex production process within a *technical efficiency* framework, suitable for public firms production processes, as the students carry out their production activity inside university, an institution that in Italy is by more than 90% government-owned. In order to express as the inputs of the process the characteristics of the students that are represented by categorical variables (such as capabilities, socio-economic status, gender, educational experience gained at high school) and that can hardly affect the results, in this paper we propose the utilization of MCA as the first stage in efficiency evaluation. To derive the measure of efficiency, DEA has been chosen as the graduate produces him/herself by utilizing multiple inputs difficult to model via a suitable production function and as it allows to properly consider the hierarchical structure of the data (as in Figure 1) and to obtain a decomposition of the efficiency score (Thanassoulis and Portela, 2002).

3. EFFICIENCY EVALUATION PROCEDURE: MCA AND DEA TWO-STAGE APPROACH

One of the most frequent problems in DEA is the lack of differentiation between the production units, which can be caused by an excessive number of inputs with respect to the total number of units in the analysis. One way of dealing with this problem, developed first by Golany and Adler (2001), is a combination of Principal Component Analysis (PCA), which describes a matrix of data through a reduced number of variables or principal components, and DEA.

Similarly to PCA for metric data, MCA (Greenacre, 1984) aims to reduce dimensionality with the least possible loss of information, whereas the interpretation of the extracted dimensions is based on the categories of the analysed variables. By using MCA on a set of variables describing the educational production process, one can both reduce the number of inputs and directly introduce the students' characteristics along with the other inputs into the subsequent efficiency evaluation by means of a quantitative measure of them expressed by the factor loadings.

Since MCA can get some negative figures and as inputs (outputs) of a DEA need to be strictly positive, an affined transformation of the data can be utilized which does not affect the results when using the Banker, Charnes and Cooper (BCC) (1984) output oriented model. Indeed, Pastor (1996) proved that the BCC output oriented model is input translation invariant and vice versa. Consequently, all MCA input data used in the output oriented DEA have been increased by the highest negative value in the vector plus one, thus ensuring strictly positive data. Let n denote the number of graduates, indexed over j such that $j = 1,\dots,n$; x_{ij} denotes the ith input used by the graduate to produce him/herself, and y_j denotes the level of the output yielded by the graduate (for instance, the final score). The production possibility set of the BCC model is defined as:

$$P_{BCC} = \left\{ (\mathbf{x}, \mathbf{y}) \mid \mathbf{x} \geq \mathbf{X}\lambda,\ \mathbf{y} \leq \mathbf{Y}\lambda, \mathbf{e}\lambda = 1,\ \lambda \geq 0 \right\} \tag{1}$$

where $\mathbf{X} = \left(\mathbf{x}_j\right) \in R^{s \times n}$, $\lambda \in R^n$ and $\boldsymbol{e}$ is a row vector with all elements equal to 1.

The condition $\boldsymbol{e}\lambda = \sum_{j=1}^{n} \lambda_j = 1$ together with $\lambda_j \geq 0$, $\forall\, j$, imposes a convexity condition on allowable ways in which the n production units may be combined. The Farrell (1957) output-oriented technical efficiency measure for graduate j, expressed by θ_j, can be determined using a standard linear programming algorithms. The kth graduate is efficient if $\theta_k = 1$ and inefficient if $\theta_k < 1$.

Since the input data set has a twofold structure (Figure 1), we will adopt a method where efficiencies can be estimated at these two different groups levels. In fact, each student's efficiency measure obtained from DEA will incorporate a component which is the consequence of the student's own efforts and a component which is the consequence of the efficiency of the faculty attended by him. Therefore, in order to properly assess the efficiency of the graduates, it is necessary to decompose the students' efficiency measures into these two components.

To graphically show this decomposition, let's consider a hypothetical data set of students from two faculties, labelled Faculty 1 and Faculty 2 (Figure 2). The frontier of Faculty 1 consists of the lines connecting A_1, B_1 and C_1 and is called *within frontier* or, according to the terminology of Thanassoulis and Portela (2002), the graduate-within-own-faculty efficiency boundary. Similarly, Faculty 2's within frontier consists of the bold lines connecting D_2, E_2 and F_2. Finally, the boundary $A_1 B_1 F_2$ envelops all graduates and is the *overall frontier* or the *graduate-within-all-faculties efficiency boundary.*

Consequently, in the output-oriented framework, graduate G_2 has an overall efficiency HG_2/HG''. This efficiency can be decomposed in: a within efficiency HG_2/HG', which represents the proportion of efficiency obtained by student G_2 relative to the best achievement obtained by students from Faculty 2 only and is due to the student's own efforts; a between efficiency HG'/HG'', given by the distance between the within and the overall frontiers, which gives a measure of the impact of Faculty 2 on his/her performance.

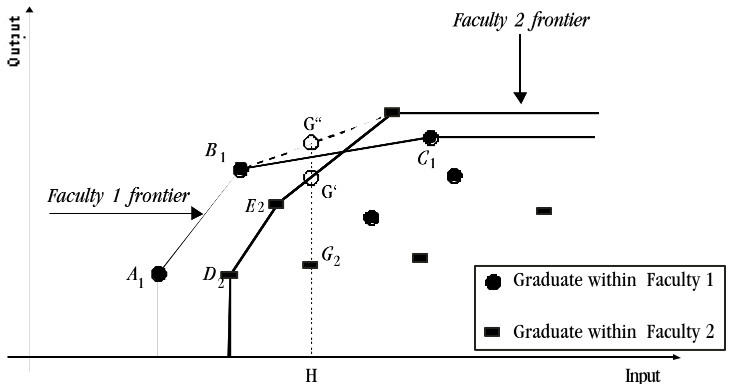


Figure 2: Decomposing Graduate's Efficiency

4. DATA AND EMPIRICAL RESULTS

The above procedure was applied to the 1998 graduates from the University of Florence. The data comes from several sources centring on an overall survey on job opportunities. Additional data were collected from the administrative archives and the Central Library. After considering only the graduates who did not move from one course program to another during their university career and checking for any possible outliers[2] we have selected the following inputs for each of the resulting 2,236 graduates: <u>a) *Human resources*</u>: 1. Average number of full and associate professors per graduate (ANFAP); 2. Average number of researchers per graduate (ANR); <u>b) *Capital resources*</u>: 3. Average number of seats in lecture halls per student (ANS); 4. Average number of lecture halls per student (ANH); 5. Average number of books in the library per student (ANB); 6.average number of journals and reviews in the library per student (ANJ); 7. Average number of university pieces of furniture per student (ANF); 8. Average number of university pieces of equipment (ANE); <u>c) *Individual inputs*</u>: 9. Gender; 10. Residence, that is macro-region of residence (three categories); 11. Type of secondary school attended (five categories); 12. Final high-school mark (HSM); 13. Regularity at school, that is if the student has repeated a year at school (two categories); 14. Working status while attending university (two categories)[3].

Starting from an interesting suggestion by Checchi (2000), we have defined as *output* of the production process an indicator of student's performance (PERF) obtained by taking the average grade at the exams divided by the relative length of study, i.e., the effective length of study compared to the obligatory length[4]. Table 1 reports the average value of the quantitative variables by faculty.

Table 1. Average Variable Values by Faculty

Faculty	ANFAP	ANR	ANS	ANH	ANB	ANJ	ANF	ANE	HSM	PERF
Agricultural Sciences	0.057	0.026	0.576	0.011	83.59	0.22	1.963	1.222	46.5	17.49
Architecture	0.011	0.008	0.195	0.002	3.86	0.034	0.357	0.276	46.7	14.85
Economics	0.013	0.008	0.241	0.003	31.87	0.129	0.484	0.084	49.5	14.12
Pharmaco-logy	0.042	0.023	0.39	0.005	2.08	0.04	1.326	1.1	48.3	16.91
Law	0.007	0.007	0.193	0.002	38.39	0.141	0.244	0.037	48.8	14.82
Engineering	0.023	0.010	0.402	0.005	4.98	0.091	0.641	0.509	52.7	17.39
Humanities	0.023	0.019	0.275	0.005	160.23	0.336	0.375	0.119	49.2	14.83

2 There are several approaches in DEA models to solve the problem of detecting influential observations (Pastor, Ruiz and Sirvent, 1999). However, in this context, the nature of the data suggests to directly check for outliers with respect to length of study (X) using the so-called three-sigma rule, that the probability is less than 5% that X is more than three standard deviations from the mean of the population. By applying this rule we deleted 41 observations.

3 It should be noted that the student's characteristics considered have been largely determined by the availability of data.

4 Indeed, according to the Italian university system, a student obtains a degree when he/she has passed a predetermined number of exams (with a minimum mark of 18 out of 30) and defended a final dissertation. It is reasonable to assume that the student who produces himself/herself as a graduate wants to finish university studies and get the degree within the institutional time established for the course program he/she attends. But, since there is a minimum number of years of enrolment but not a maximum, the actual length of study time often exceeds the institutional target.

Faculty	ANFAP	ANR	ANS	ANH	ANB	ANJ	ANF	ANE	HSM	PERF
Medicine and Surgery	0.136	0.050	1.708	0.02	88.05	0.154	2.461	1.863	50.5	21.18
Biology and Mathematics	0.057	0.027	0.828	0.011	116.68	0.471	1.492	1.192	48.9	17.17
Political Sciences	0.011	0.008	0.18	0.002	36.16	0.138	0.35	0.107	46.9	14.91
Sciences of Education	0.013	0.011	0.256	0.004	40.26	0.106	0.583	0.093	46.6	16.63
Totals	**0.023**	**0.013**	**0.354**	**0.005**	**48.76**	**0.158**	**0.648**	**0.367**	**48.6**	**15.66**

It can be noted that there are strong differences in the availability of resources among faculties which reflect clear differences in the way the graduate production process is implemented within faculties. Consequently, the importance of comparing "like-to-like" becomes an overriding consideration in our achievement of a "fair" evaluation scheme.

After classifying continuous variables into classes, we have applied a MCA on the above 14 variables using the SPAD 3.01 software. The total variance expressed by the first two factors is 24.6%: a good result if we think to the original dimension of vector space, that is 69-14=55, with total inertia equal to (69/14)-1=3.93 (Lebart, Morineau and Warwick, 1984). It is possible to locate the correspondence between two or more variables, between variables and graduates and to state roughly a range along the first factor of all the faculty resources, both human and capital, while the second factor better describes the characteristics of the graduates[5].

As a matter of fact, when the values of the first factor do increase from the right to the left, one can see an increase in number of professors, of lecture halls, of textbooks, etc, whereas as the of the second factor do increase, one gradually moves from student characterized by low high school marks, living long way from Florence university to students who didn't work during their academic career until one reaches graduates living in Florence, who performed well at high school, and who worked during their university studies. It should be stressed that the MCA scores (Table 2) show the same faculty ordering as the one in Table 1.

The measures of output-efficiency for each of the 2,236 graduates, have been obtained by using the DEAP version 2.1 (Coelli, 1996). The input data is in the form of MCA scores of Table 2. Table 3 below shows that 14 graduates, out of 2,236, are identified with 1.0 score 1.0, as being fully efficient. The average overall efficiency score is 0.575. This means that on average, inefficient graduates should be able to increase outputs by 42.5% without having to increase inputs. In other words, the graduates below the overall envelope, that is, with efficiency scores less than 1, can

5 The MCAA results are available from the authors upon request.

achieve a higher performance with the same level of input. The summary across all students conceals some potentially interesting variations in efficiencies among students from different faculties (Table 3). Let's consider for example the Faculty of Pharmacology. The graduates from this faculty have a relatively poor between efficiency score (0.793) compared to the average of the population (0.876).

A closer look reveals that the within efficiency measure is very high compared with the overall efficiency (see Figure 3a).

Table 2. Average Variable Values by Faculty

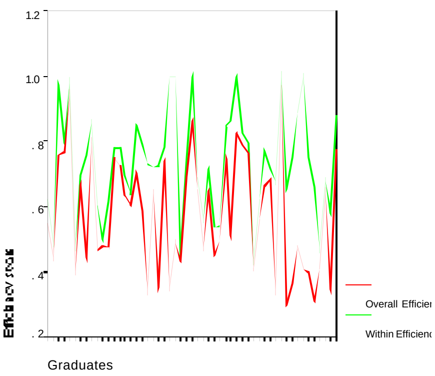
Faculty	Factor 1			Factor 2		
	Mean	**Min**	**Max**	**Mean**	**Min**	**Max**
Agricultural Sciences	3.16	3.09	3.22	2.18	2.10	2.26
Architecture	1.29	1.19	1.36	1.82	1.57	2.00
Economics	1.22	1.11	1.38	3.19	3.05	3.27
Pharmacology	2.51	2.34	2.60	2.17	2.08	2.31
Law	1.10	1.00	1.17	1.16	1.00	1.36
Engineering	2.04	1.94	2.16	2.74	2.63	2.94
Humanities	1.86	1.79	2.01	3.00	2.68	3.10
Medicine and Surgery	3.08	2.94	3.19	2.09	1.93	2.28
Biology and Mathematics	3.50	3.41	3.55	1.94	1.85	2.04
Political Sciences	1.25	1.17	1.34	2.20	2.05	2.32
Sciences of Education	1.60	1.42	1.73	3.07	2.63	3.29
	1.73	**1.00**	**3.55**	**2.36**	**1.00**	**3.29**

Thus, the graduates of Pharmacology are performing well within the faculty – they are performing on or close to the faculty's own efficiency frontier - but are prevented from better performing due to faculty's relative inefficiency, that is the faculty's efficiency frontier is well inside the overall efficiency frontier. Let's now consider the Faculty of Agriculture, which has a relatively high between efficiency score (0.866). As can be noted from Figure 3b, the within and the overall efficiency measures are quite similar. Hence, the graduates of Agriculture are not performing particularly well within the faculty and the main constraint on achieving higher efficiency is their own efforts, as the faculty's efficiency frontier is close to the overall one.

Table 3. Summary of Efficiency Measures by Faculty

		Overall efficiency				Within efficiency				Between efficiency			
Faculty	**N. of units**	**Mean**	**Min**	**Max**	**Efficient units**	**Mean**	**Min**	**Max**	**Efficient units**	**Mean**	**Min**	**Max**	**Efficient units**
Agriculture	70	**0.592**	0.239	0.963	0	**0.700**	0.254	1.000	11	**0.866**	0.260	0.965	0
Architecture	396	0.558	0.262	0.990	0	0.629	0.269	1.000	8	0.903	0.651	0.990	0
Economics	388	0.533	0.248	1.000	1	0.659	0.291	1.000	7	0.823	0.472	1.000	6
Pharmacology	53	**0.577**	0.300	0.961	0	**0.736**	0.437	1.000	6	**0.793**	0.356	0.961	0
Law	275	0.683	0.345	1.000	8	0.685	0.345	1.000	8	0.999	0.990	1.000	188
Engineering	214	0.584	0.216	0.929	0	0.731	0.282	1.000	11	0.799	0.271	0.930	0
Humanities	230	0.499	0.226	1.000	1	0.641	0.323	1.000	8	0.783	0.276	1.000	8
Medicine& Surg.	80	0.722	0.225	1.000	1	0.731	0.225	1.000	5	0.989	0.789	1.000	6
Biology& Math.	170	0.590	0.230	1.000	1	0.671	0.238	1.000	12	0.890	0.250	1.000	10
Political Sciences	164	0.556	0.265	0.897	0	0.713	0.406	1.000	9	0.782	0.336	0.897	0
Sciences of Educ.	196	0.562	0.244	1.000	2	0.565	0.245	1.000	5	0.995	0.753	1.000	32
	2236	**0.575**	**0.216**	**1.000**	**14**	**0.664**	**0.225**	**1.000**	**90**	**0.876**	**0.250**	**1.000**	**250**

The above two faculties can also obtain helpful information from these individual level results together with the students' characteristics emerging from Factor 2 scores. Indeed, the students of both faculties present similar characteristics (i.e. they live in Florence, didn't work during their university studies, got high school marks).



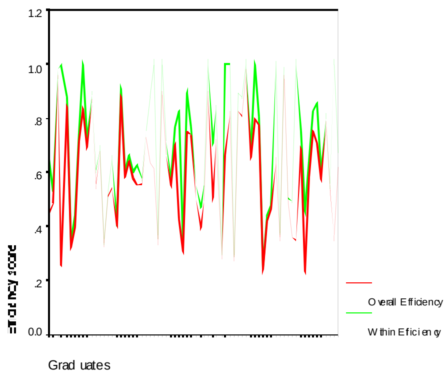


Figure 3a: *Pharmacology* ***Figure 3b:*** *Agricultural Sciences*

In order to increase efficiency, Pharmacology needs to find ways of increasing its own efficiency by moving its own frontier out, with better management of "program" inputs (e.g. teaching and other staff, student workload, etc). On the other hand, Agriculture needs to find ways of stimulating students' efforts in order to increase both within and overall efficiencies, getting its students performing closer to individual and overall efficiency frontiers.

5. CONCLUSIONS

The aim of this paper was to propose the use of MCA in order to include the student's characteristics and to reduce the number of inputs for a subsequent DEA. We run several DEA models in order to check the sensitivity of results to the number of MCA factors, with quite similar results.

Thus, a reduction of the data describing the university production process through MCA can allow to take account of more information than would otherwise be the case. Furthermore, the individual DEA approach used here has disentangled the effect of the individual and the effect of the faculty attended in determining technical efficiency. The results obtained can identify for each faculty whether they need to stimulate their students' efforts by better management of individual inputs (e.g. improve the student cohesion) or whether they need to increase efforts to better management of faculty inputs (e.g. teaching and other staff). Indeed, it is possible that a different strategy is required, depending on whether or not the individual is efficient within his own faculty. Further investigation is needed on how faculties might best achieve these aims.

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M|G|∞ QUEUE BUSY CYCLE RENEWAL FUNCTION FOR SOME PARTICULAR SERVICE TIME DISTRIBUTIONS

MANUEL ALBERTO M. FERREIRA

Abstract In this paper we present formulas to compute the busy cycle renewal function for the $M|G|\infty$ queue, considering service time distributions that arise when we study parameters related to the busy period. The busy cycle renewal function in t gives the mean number of busy periods that begin in $[0, t]$.

Keywords: M | G | ∞, busy cycle, renewal function.

1. INTRODUCTION

In the $M|G|\infty$ queueing system, λ is the customers Poisson process arrival rate, each customer gets a service that is a positive real value with distribution function $G(\cdot)$ and mean α, being $\alpha = \int_0^\infty [1-G(t)]\,dt$; there are infinite servers and the service time of each customer is independent of the others. The traffic intensity is $\rho = \lambda\alpha$.

In a queueing system we usually call busy period a period that begins when a customer arrives there, and it is empty; it ends when a customer leaves the system letting it empty, and there is always at least one customer present. So, in a queueing system, there is always a sequence of idle periods and busy periods.

Let's consider the $M|G|\infty$ system with time origin at the beginning of a busy period. The instant $0, t_1, t_2, \ldots$ at which a busy period begins, are the arrival epochs of a renewal process (Takács (1962)). We say that a cycle is complete when a renewal occurs, that is, a busy period begins. These cycles are busy cycles; and their length is a real value that we will call Z.

So

$$Z = B + I \qquad (1.1)$$

where B is the busy period length and I the idle period one.

Takács (1962) proved that B and I are stochastically independent and, still, that the Z Laplace-Stieltjes transform, $\overline{Z}(s)$, is given by

$$\overline{Z}(s) = 1 - \frac{1}{(s+\lambda)P_{00}(s)} \qquad (1.2),$$

where $P_{00}(s)$ is the $p_{00}(t)=e^{-\lambda\int_0^t[1-G(v)]dv}$ Laplace-Stieltjes transform, probability of the system emptiness at t having been initially empty.

That author showed also that

$$E[Z]=\frac{e^{\rho}}{\lambda} \tag{1.3}$$

and

$$E\left[Z^2\right]=2\lambda^{-1}e^{2\rho}\int_0^{\infty}\left(e^{-\lambda\int_0^t[1-G(v)]dv}-e^{-\rho}\right)dt+2\lambda^{-2}e^{\rho} \tag{1.4}$$

Being I exponentially distributed with parameter λ its Laplace-Stieltjes transform is $\bar{I}(s)=\frac{\lambda}{\lambda+s}$ and the ratio $\frac{\bar{Z}(s)}{\bar{I}(s)}$ gives the expression $\bar{B}(s)=1+\frac{1}{\lambda}\left[s-\frac{1}{P_{00}(s)}\right]$ for the Laplace-Stieltjes transform of B (Stadje (1985)), whose inversion is a complex problem except for some service distributions (see Ferreira (1991), (1995), (1998)).

Our work in this paper will focus on the $M|G|\infty$ queue busy cycle, in particular in its renewal function study.

2. THE M|G|∞ QUEUE BUSY CYCLE RENEWAL FUNCTION

The renewal function, R of a renewal process is given for $R=1+F+F^{*2}+F^{*3}+\ldots$ where F^{*n} is the n-th interrenewal time convolution with itself distribution function (see, for instance, Çinlar (1975)). It gives the mean number of renewals in $[0,t]$. For instance, in the application of this model to unemployment situations, a busy period is a period of unemployment. And, in illness situations, a busy period is an epidemic period. See, about this kind of applications Ferreira (2003 and 2003a).

To compute the $M|G|\infty$ queue busy cycle renewal function we have, using the Laplace-Stieltjes transform, $\bar{R}(s)=\frac{1}{s}+\frac{1}{s}\bar{Z}(s)+\frac{1}{s}\bar{Z}^2(s)+\ldots+\frac{1}{s}\bar{Z}^n(s)=\frac{s^{-1}}{1-\bar{Z}(s)}$

$$=\frac{s^{-1}}{1-1+\frac{1}{(s+\lambda)P_{00}(s)}}=\frac{(s+\lambda)P_{00}(s)}{s}=P_{00}(s)+\lambda\frac{1}{s}P_{00}(s).$$

So, $R(t) = p_{00}(t) + \lambda\int_0^t p_{00}(u)\,du$, and:

$$R(t) = e^{-\lambda\int_0^t [1-G(v)]dv} + \lambda\int_0^t e^{-\lambda\int_0^t [1-G(v)]dv}\,du \quad (2.1).$$

Note that:

$$\lim_{t\to\infty}\left[R(t) - \frac{\lambda}{e^{\rho}}t\right] = \lim_{t\to\infty}\left[e^{-\lambda\int_0^t [1-G(v)]dv} + \lambda e^{-\rho}\int_0^t e^{\rho-\lambda\int_0^u [1-G(v)]dv}\,du - \frac{\lambda}{e^{\rho}}t\right] =$$

$$= e^{-\rho} + \lambda e^{-\rho}\lim_{t\to\infty}\int_0^t \left(e^{\rho-\lambda\int_0^u [1-G(v)]dv} - 1\right)du = e^{-\rho} + \lambda e^{-\rho}\int_0^\infty \left(e^{\lambda\int_u^\infty [1-G(v)]dv} - 1\right)du$$

So, it is easy to see that:

$$\lim_{t\to\infty}\left[R(t) - \frac{t}{E[Z]}\right] = \frac{VAR[Z] + E^2[Z]}{2E^2[Z]} \quad (2.2)$$

as it has to be with a renewal function.

– As $e^{-\rho} \le p_{00}(t) \le 1$,

$$p_{00}(t) + \lambda e^{-\rho}t \le R(t) \le p_{00}(t) + \lambda t \quad (2.3)$$

and, still,

$$e^{-\rho}(1+\lambda t) \le R(t) \le 1 + \lambda t \quad (2.4),$$

we conclude that

$$\lim_{\alpha\to 0} R(t) = 1 + \lambda t \quad (2.5),$$

as it has to be because, when the service time is nul, when it arrives each customer begins a busy period.

And the arrival instants, in the $M|G|\infty$ system, occur according to a Poisson process at rate λ.

$$-\frac{d}{dt}R(t) = p_{00}(t)\{-\lambda[1-G(t)]\} + \lambda p_{00}(t) = \lambda G(t)\,p_{00}(t) \ge 0.$$

So $R(t)$ increases with t.

3. R(t) VALUES FOR SOME PARTICULAR SERVICE TIME DISTRIBUTIONS

A. The $M|G|\infty$ emptiness probability at time t, as time function, being the initial instant the one of the beginning of a busy period (at which a customer arrives at the system finding it empty) is determined by the sign of $\frac{g(t)}{1-G(t)}-\lambda G(t),\ t\geq 0$ (Ferreira (1996)) where $g(\cdot)$ and $G(\cdot)$ are, respectively the service time p.d.f. and d.f. .

Putting $\frac{g(t)}{1-G(t)}-\lambda G(t)=\beta(t)$ ($\beta(\cdot)$ is any time function) we get

$$\frac{dG(t)}{dt}=-\lambda G^2(t)-[\beta(t)-\lambda]\,G(t)+\beta(t) \quad (3.1)$$

that is a Ricatti equation about $G(\cdot)$.

Solving it, after noting that $G(t)=1,\ t\geq 0$ is a solution, we get

$$G(t)=1-\frac{1}{\lambda}\frac{\left(1-e^{-\rho}\right)e^{-\lambda t-\int_0^t \beta(u)du}}{\int_0^\infty e^{-\lambda w-\int_0^w \beta(u)du}dw-\left(1-e^{-\rho}\right)\int_0^t e^{-\lambda w-\int_0^w \beta(u)du}dw},$$

$$t\geq 0,\ -\lambda\leq\frac{\int_0^t \beta(u)\,du}{t}\leq\frac{\lambda}{e^{\rho}-1} \quad (3.2).$$

Putting directly (3.2) in (2.1) we get the corresponding value for $R(t)$. If $\beta(t)=\beta$ (constant)

$$G(t)=1-\frac{\left(1-e^{-\rho}\right)(\lambda+\beta)}{\lambda e^{-\rho}\left(e^{(\lambda+\beta)t}-1\right)+\lambda},\quad t\geq 0,\quad -\lambda\leq\beta\leq\frac{\lambda}{e^{\rho}-1}\ \text{(Ferreira (1998))} \quad (3.3).$$

For this service time distributions collection the busy period length is exponentially distributed with an atom at the origin.

After (2.1) we get $R(t)=e^{-\rho}(1+\lambda t)+\left(1-e^{-\rho}\right)\frac{\beta}{\lambda+\beta}e^{-(\lambda+\beta)t}+\left(1-e^{-\rho}\right)\frac{\lambda}{\lambda+\beta}$, $-\lambda<\beta<\frac{\lambda}{e^{\rho}-1}$. It is easy to show that $R(t)-\lambda e^{-\rho}-\frac{VAR[Z]+E^2[Z]}{2E^2[Z]}=0,\ t\geq 0.$

For the collection given by (3.2) we can show that

$$R(t) \geq \frac{(1+\lambda t)+\left(1+e^{-\rho}\right)\left(1+e^{-\frac{\lambda}{1-e^{-\rho}}t}\right)}{e^{\rho}}, \quad t \geq 0, \quad -\lambda \leq \frac{\int_0^t \beta(u)du}{t} \leq \frac{\lambda}{e^{\rho}-1} \qquad (3.4)$$

B. The $M|G|\infty$ populational process, with time origin at a busy period beginning instant, mean behaviour as time function determinated by the sign of $\frac{g(t)}{1-G(t)}-\lambda,\ t\geq 0$ (Ferreira (2003 and 2003a)).

Putting $\frac{g(t)}{1-G(t)}-\lambda=\beta(t)$ ($\beta(\cdot)$ is any time function) we get

$$G(t)=\{1-[1-G(0)]\}e^{-\lambda t-\int_0^t \beta(u)\,du}, \quad t\geq 0, \quad \frac{\int_0^t \beta(u)\,du}{t}\geq -\lambda \qquad (3.5).$$

Putting directly (3.5) in (2.1) we get the corresponding value for $R(t)$.

For $\beta=0$, (3.5) becomes

$$G(t)=1-[1-G(0)]e^{-\lambda t}, \quad t\geq 0 \qquad (3.6)$$

with $1-G(0)=\rho$. In this situation the $M|G|\infty$ populational process, with time origin at a busy period beginning instant, mean is constant with value rho (Ferreira (2003 and 2003a)). As for $R(t)$ we have

$$R(t)=e^{-\rho\left(1-e^{-\lambda t}\right)}+\lambda\int_0^t e^{-\rho\left(1-e^{-\lambda u}\right)}du \qquad (3.7)$$

and also

$$e^{-\rho\left(1-e^{-\lambda t}\right)}+\lambda e^{-\rho}t \leq R(t) \leq e^{-\rho\left(1-e^{-\lambda t}\right)}+\lambda t \qquad (3.8)$$

CONCLUSIONS

After a short study about $R(t)$ and its properties, we present the formulas for it in the situation of particular distribution functions related to important busy period parameters behaviour study as time functions.

We show also bounds that help, in this case, when the formulas are note so friendly.

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MODELING OF CONTRACTS IN SUPPLY CHAIN MANAGEMENT

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Abstract Recent years have seen a growing interest among researchers and practitioners in the field of supply chain management. When one or more parties of the supply chain try to optimize their own profits, system performance may be hurt. Supply chain contract is a coordination mechanism that provides incentives to all of its members so that the decentralized supply chain behaves nearly or exactly the same as the integrated one.

Keywords: supply chain, contracts, coordination

1. INTRODUCTION

Supply chain is defined as a system of suppliers, manufacturers, distributors, retailers and customers where material, financial and information flows connect participants in both directions.

Supply chain management is now seen as a governing element in strategy and as an effective way of creating value for customers. An increasing number of companies in the world subscribe to the idea that developing long-term coordination and cooperation can significantly improve the effectiveness of supply chains and provide a way to ensure competitive advantage. The expanding importance of supply chain integration presents a challenge to operations research to focus more attention on supply chain modeling.

Double marginalization (Spengler, 1950) is a well-known cause of supply chain inefficiency. Double marginalization problem occurs whenever the supply chain's profits are divided among two or more firms and at least one of the firms influences demand. Each firm only considers its own profit margin and does not consider the supply chain's margin.

Developing strategies to decrease the risk faced by the retailer is becoming more and more critical in a supply chain, especially in the global marketplace where firm-to-firm competition is being replaced by supply-chain-to-supply-chain competition. Among the solutions, supply chain contracts, which have drawn much attention from the researchers recently, are used to provide some incentives to adjust the relationship of supply chain partners to coordinate the supply chain, i.e., the total profit of the decentralized supply chain is equal to that achieved under a centralized system. The format of supply chain contracts varies in and across industries.

When the demand is stochastic with the newsvendor model can be applied. The newsvendor model is not complex, but it is sufficiently rich to study three important questions in supply chain coordination.

1. Which contracts coordinate the supply chain? A contract is said to coordinate the supply chain if the set of supply chain optimal actions is a Nash equilibrium, i.e., no firm has a profitable unilateral deviation from the set of supply chain optimal actions. Ideally, the optimal actions should also be a unique Nash equilibrium, otherwise the firms may “coordinate” on a sub-optimal set of actions. In the newsvendor model the action to coordinate is the retailer’s order quantity.

2. Which contracts have sufficient flexibility to allow for any division of the supply chain’s profit among the firms? If a coordinating contract can allocate rents arbitrarily, then there always exists a contract that Pareto dominates a non-coordinating contract, i.e., each firm’s profit is no worse off and at least one firm is strictly better off with the coordinating contract.

3. Which contracts are worth adopting? Although coordination and flexible rent allocation are desirable features, contracts with those properties tend to be costly to administer. As a result, the contract designer may actually prefer to offer a simple contract even if that contract does not optimize the supply chain’s performance. A simple contract is particularly desirable if the contract’s efficiency is high and if the contract designer captures the significant share of supply chain profit.

The aim of this paper is to analyze and to compare different types of contracts and to seek for a general framework that synthesizes existing results for a variety of supply chain contract forms.

2. BASIC MODEL

We consider a supply chain in one-period setting in which a supplier sells to a retailer facing stochastic demand from consumers. We assume that stochastic demand x has a continuous distribution $F(x)$ that is invertible. The demand distribution and cost information are common knowledge. We define the following quantities:

q retailer’s total order quantity;

c supplier’s production cost;

r retail price;

s salvage value.

The setting can be characterized as a newsvendor problem.

Centralized solution

Centralized solution is a benchmark for the decentralized supply chain. The centralized chain is considered as an integrated firm that controls manufacturing and sales to consumers. The profit of an integrated firm for stocking level q is

$$z(q) = (r - c)q - (r - s)\int_0^q F(x)\, dx.$$

The problem is concave in q and the optimal solution is given by

$$q^0 = F^{-1}\left(\frac{r-c}{r-s}\right)$$

The maximum system profit $z(q^0)$ is completely determined by the stocking level q^0. Decentralized solution can be improved by contracting. The contract coordinates the chain if it induces the choice of the centralized system's optimal stocking level q^0.

3. WHOLESALE PRICE CONTRACTS

With a wholesale price contract the supplier charges the retailer w per unit purchased. The retailer faces a problem analogous to that of the integrated chain with the same salvage opportunities. The principal difference is that the retailer must buy stock at the wholesale price w instead of producing it at cost c.

The retailer's profit is

$$z_R(q) = (r-w)q - (r-s)\int_0^q F(x)\,dx.$$

The retailer's problem is concave in q and the optimal solution is given by

$$q(w) = F^{-1}\left(\frac{r-w}{r-s}\right)$$

The supplier acts as a Stackelberg leader and anticipates how the retailer will order for any wholesale price.

The supplier anticipates a demand curve $q(w)$ and the profit

$$z_S(w) = (w-c)q(w) = (w-c)F^{-1}\left(\frac{r-w}{r-s}\right)$$

The supplier knows exactly what retailer will order at every wholesale price and bears no responsibility for the product. All uncertainty regarding supply profits is foisted onto the retailer. The wholesale price contract coordinates the chain only if the supplier earns a non-positive profit. So the supplier clearly prefers a higher wholesale price. As a result, the wholesale price contract is generally not considered a coordinating contract. The richer contracts differ from wholesale price contracts by allowing the supplier to assume some of the risk arising from stochastic demand. As an example we introduce buy back contracts.

4. BUY BACK CONTRACTS

With a buy back contract (Pasternack, 1985) the supplier charges the retailer w per unit purchased, but pays the retailer b per unit remaining at the end of the season.

A retailer should not profit from left over inventory, so assume $b \le w$. There is assumed that a returns policy on the decentralized chain introduces no additional cost beyond that incurred by the centralized system.

The retailer's profit is

$$z_R(q) = (r - w)q - (r - b)\int_0^q F(x)\, dx.$$

The retailer still faces a newsvendor problem. The optimal solution is

$$q(w, b) = F^{-1}\left(\frac{r - w}{r - b}\right)$$

No returns or full returns are suboptimal. An intermediary policy results in chain coordination. The supplier offers a contract $(w(e),\ b(e))$ for $\varepsilon \in (0, r - c\rangle$ where

$$w(\varepsilon) = r - \varepsilon,\ \ b(\varepsilon) = r - \frac{\varepsilon(r - s)}{r - c}.$$

For all $\varepsilon \in (0, r - c\rangle$,

$$r > w(e) > b(e) \ge s \text{ and}$$

$$\frac{r - w(\varepsilon)}{r - b(\varepsilon)} = \frac{r - c}{r - s}.$$

The retailer orders the integrated chain quantity

$$q(w(e),\ b(e)) = q^0$$

and system profit is equal to the integrated chain profit $z(q^0)$.

Retailer's profit is increasing in e

$$z_R[w(\varepsilon),\ b(\varepsilon)] = \frac{\varepsilon}{r - c} z(q^0).$$

Supplier's profit is decreasing in e

$$z_s[w(\varepsilon),\ b(\varepsilon)] = \left(1 - \frac{\varepsilon}{r - c}\right) z(q^0).$$

5. OTHER TYPES OF CONTRACTS

Quantity flexibility contracts define terms under which the quantity a retailer ultimately orders from the manufacturer may deviate from a previous planning estimate (Tsay, 1999 and Larievier,1999).Unlike buy buck contracts which focus on flexibility in adjusting price, quantity flexibility contracts focus on flexibility in adjusting ordering quantity. The basic idea is that when a retailer places an initial order q, the supplier agrees to provide up to $(1 + u)q$ units to the system. At the same time the retailer commits to order at least $(1 - d)q$ units. After observing the demand for a short period, the retailer can decide to order any quantity between $(1 - d)q$ and $(1 + u)q$ at the wholesale price w. Quantity flexibility contracts can lead to a much more profit of the decentralized supply chain than that achieved without the contracts.

Backup agreements (Eppen and Iyer, 1997) state that if a retailer commits to a number of units for the season, the supplier will hold back a fraction of the commitment and the retailer can order up to this backup quantity at the original purchase price after observing early demand. A backup agreement states that if the retailer commits to a number of units for the season, the manufacturer holds back a constant fraction *?* of the commitment q and delivers the remaining units (1 -*?*)q at the beginning of the selling season. After observing early demand, the retailer can order up to this backup quantity for the original purchase cost and receive quick delivery but will pay a penalty cost p for any of the backup units it does not buy.

Option contracts (Barnes-Schuster *et a*l., 2002) specify that in addition to a firm order at a regular price, the retailer can also purchase options at an option price at the beginning of the selling season. After observing early demand, the retailer can choose to exercise those options at an exercise price. The retailer makes a firm order q at the beginning of the selling season at a wholesale price w. In addition, he purchases n options at an option price w_o. In the second period, the retailer may choose to exercise options at an exercise price w_e. Options provide flexibility to a retailer to respond to market changes in the second period quickly.

Price protection (Lee *et a*l. 2000) states that the supplier pays the retailer a credit applying to the retailer's unsold goods when the wholesale price drops during the life cycle. The basic idea of the model is that the retailer orders q products from the supplier at the beginning of the first period at a wholesale price w_1. At the beginning of the second period, the wholesale price of the same product drops to w_2 because of the introduction of new products. To share the risk of the retailer, the manufacturer will pay a rebate credit b to the retailer for all unsold inventory at the end of the first period. It is similar to a buyback contract, but looking at the dynamic optimal price protection policy when the product in the markets faced with obsolescence during multiple periods.

6. CONCLUSION

There is a vast literature on supply chain contracts recently. However, little work has been done on the relationships of those supply chain contract models. The analysis of the simple cases of contracts gives recommendations for more complex real problem. Real problems in supply chains are solved by joint problem solving in supply chain partnership. The partnership relations are based on supply contracts.

The research of a general framework that synthesizes existing results for a variety of supply chain contract forms would be very desirable.

The re search pro ject was sup ported by Grant No. 402/01/0771 from the Grant Agency of Czech Republic „Modeling of Supply Chains“ and CEZ: J 18/98: 311401001 from the Uni ver sity of Eco nom ics „Models and Methods for Eco nomic Decisions“.

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A QUANTITATIVE APPROACH TO MUTUAL DEBT COMPENSATION PROBLEM

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Abstract The article deals with the problem of secondary financial insolvency. The problem of mutual debt minimisation is formulated as a maximum flow problem in the debt digraph. The article considers a problem of controlled revitalization of the firms, too.

1. INTRODUCTION

The mutual debt compensation problem (MDC-P) was described by Fecenko [4] for the first time. He proposed a solution of the MDC-P as a circulation maximising. He also proposed a method of mutual debt minimisation using capital subsidiaries. Gazda [5], [6] proposed an alternative method of this problem using subsidiaries and investigated the alternatives of MDC-P with possibility of debt structure changes.

2. DEBT DIGRAPH

Let $V = \{1, 2, \ldots, n\}$ be the set of firms involved in the mutual debt compensation process. Let y_{ij} be the debt which firm i owes towards firm j. We assume that $0 \le y_{ij}$ and at least one of equations $y_{ij} = 0$, $y_{ji} = 0$ holds. Consider a creditor-debtor relation $E \subseteq V \times V$ defined as follows:

$$E = \{ (i, j) \mid y_{ij} > 0 \} \tag{1}$$

Let $y: E \to R^+$ be a positive real value function of the debt level of each creditor-debtor pair $(i, j) \in E$. That means

$$y(e) = y_{ij} \quad \text{for every } e \in E, \text{ where } e = (i, j) \tag{2}$$

The debt digraph is the weighted directed graph $G = (V, E, y)$. Assign

$$I_i^+ = \{ j \mid j \in V, (j, i) \in E \}, \qquad I_i^- = \{ j \mid j \in V, (i, j) \in E \}. \tag{3}$$

I_i^+ is the set of creditors of the firm i, I_i^- is the set of debtors of the firm i.

The balance $b(i)$ of each firm i is

$$b(i) = \sum_{j \in I_i^+} y_{ji} - \sum_{j \in I_i^-} y_{ij}. \tag{4}$$

The first sum $\sum_{j\in I_i^+} y_{ji}$ in (4) represents receivables of the i-th firm and the second sum $\sum_{j\in I_i^-} y_{ij}$ in (4) are payables (or debts) of the firm i.

Lets denote

$$b^+(i) = \max\{0,\ b(i)\}, \qquad b^-(i) = \max\{0,\ -b(i)\} \tag{5}$$

Since $y_{ij} = 0$ for $(i,j)\notin E$, it holds $\sum_{j\in V-I_i^+} y_{ji} = 0$ and hence

$$\sum_{j=1}^{n} y_{ji} = \sum_{j\in V} y_{ji} + \sum_{j\in V-I_i^+} y_{ji} = \sum_{j\in I_i^+} y_{ji} \tag{6}$$

For the same reason we can write

$$\sum_{j=1}^{n} y_{ij} = \sum_{j\in V} y_{ij} = \sum_{j\in I_i^-} y_{ij} \tag{7}$$

Since

$$\sum_{j=1}^{n} b(i) = \sum_{j=1}^{n}\left(\sum_{j\in I_i^-} y_{ji} - \sum_{j\in I_i^+} y_{ij}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} y_{ji} - \sum_{i=1}^{n}\sum_{j=1}^{n} y_{ij} = 0 \tag{8}$$

we have

$$\sum_{j=1}^{n} b(i) = 0 \tag{9}$$

The sum

$$\mathbf{Y} = \sum_{i=1}^{n}\sum_{j=1}^{n} y_{ij} \tag{10}$$

is called the total debt in debt digraph $G = (V, E, y)$. Two mutual debt compensation problems can be formulated:

1. To minimize the total debt $\mathbf{Y}$ by mutual debt compensation without any subsidiary.
2. To minimize the total debt $\mathbf{Y}$ with the given amount C of subsidiary.

Let's have a cycle $\mathbf{c}$ in

$$G = (V, E, y) \qquad \mathbf{c} = v_1, (v_1, v_2), v_2, \ldots, v_{k-1}, (v_{k-1}, v_k), v_k \tag{11}$$

with capacity

$$y(\mathbf{c}) = \min\{y(e) \mid e \in c\} \tag{12}$$

Presence of a cycle (11) with capacity (12) in digraph G means the existence of mutual debt cycle in the corresponding firm structure. In such a situation we can simplify the creditor-debtor relation by subtracting the cycle capacity $y(\mathbf{c})$ from the weight $y(e)$ for all edges $e \in \mathbf{c}$

$$y(e) := y(e) - y(c) \quad \text{for all edges } e \in \mathbf{c}. \tag{13}$$

The result of this operation is the reduction of the total debt in the digraph. At least one edge in cycle **c** gets a new weight $y(e)$ equal to zero. Then, the creditor – debtor pair represented by the edge with zero weight is removed from the digraph $G = (V, E, y)$. By repeating this operation we can discard all cycles in debt digraph – after the elimination of all cycles the digraph becomes acyclic.

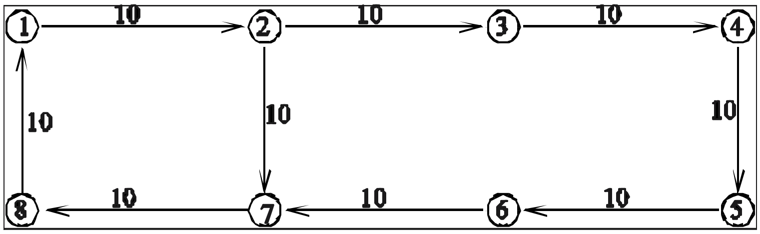

Figure 1.

As it is shown in Figure 1 it is obvious that the order of cycle elimination in digraph $G = (V, E, y)$ is decisive for maximising the total debt. If we take the cycle 1, (1,2), 2, (2,7), 7, (7,8), 8, (8,1), 1 with capacity 10, we can lower the total debt in the graph by 40 units. On contrary, if we choose the cycle 1, (1,2), 2, (2,3), 3, (3,4), 4, (4,5), 5, (5,6), 6, (6,7), 7, (7,8), 8, (8,1), 1 with the same capacity of 10 units, we can eliminate 80 debt units. In both cases the debt elimination leads to the cycle elimination – we get an acyclic digraph.

Let us denote the residual debt after cycle elimination as x_{ij}. The total residual debt can be expressed as

$$f(\mathbf{x}) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} \tag{14}$$

Since x_{ij} represents residual debts after the cycle elimination the following relation must hold

$$0 \le x_{ij} \le y_{ij} \tag{15}$$

The cycle elimination must not worsen nor make better the financial situation of each subject participating in mutual debt compensation. Hence the balance $b(i)$ defined in (4) of each firm $i \in V$ must not be influenced by mutual debt compensation, hence for every $i \in V$ the following equation must hold:

$$\sum_{j=1}^{n} x_{ji} - \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} y_{ji} - \sum_{j=1}^{n} y_{ij} = b(i) \tag{16}$$

The problem 1. of mutual debt minimization can be expressed as follows: To minimize objective function (14) subject to (15)) and (16).

Remark: From (15) it follows that $y_{ij}=0$ implies $x_{ij}=0$. Hence the total debt minimization does not create a new creditor–debtor pair. This fact can be considered as a good property of the proposed model.

3. MUTUAL DEBT COMPENSATION AS A MAX-FLOW-MIN-COST PROBLEM

Let $G=(V,E,y)$ be a debt digraph, let's denote

$$V^{+}=\{i\mid i\in V,\ b(i)>0\},\ V^{0}=\{i\mid i\in V,\ b(i)=0\},\ V^{-}=\{i\mid i\in V,\ b(i)<0\} \tag{17}$$

The sets V^{+},V^{0},V^{-} pairwise disjunctive and $V^{+}\cup V^{0}\cup V^{-}=V$. Let z, u be two elements, $z\notin V$, $u\notin V$. We define a new weighted digraph $\overline{G}=(\overline{V},\overline{E},\overline{y})$ as follows:

$$\overline{V}=V\cup\{z,u\},\ \overline{E}=E\cup\{(z,i)\mid i\in V^{-}\}\cup\{(i,z)\mid i\in V^{+}\} \tag{18}$$

$$\overline{y}(e)=\begin{cases} y(e) & \text{if } e\in E\\ b^{+}(i) & \text{if } e=(z,i), \quad \text{where } i\in V^{-}\\ b^{-}(i) & \text{if } e=(i,u), \quad \text{where } i\in V^{+}\end{cases} \tag{19}$$

We can extend the matrix $\{y_{ij}\}_{i,j\in V}$ to the matrix $\{\overline{y}_{ij}\}_{i,j\in\overline{V}}$ according to (19):

$$\overline{y}_{ij}=\begin{cases} y(e) & \text{if } e=(i,j)\in\overline{E}\\ 0 & \text{if } e=(i,j)\notin\overline{E}\end{cases} \tag{20}$$

By definitions (19) and (20) is $\overline{y}_{zz}=\overline{y}_{zu}=\overline{y}_{uz}=\overline{y}_{uu}=0$, $\overline{y}_{zi}=b^{-}(i)$ for $i\in V^{-}$ anbd $\overline{y}_{iu}=b^{+}(i)$ for $i\in V^{+}$. For $i\in V$ and $j\in V$ is $\overline{y}_{ij}=y_{ij}$. Then the equation (4) can be rewritten using (7) and just mentioned facts as follows

$$b(i)=b^{+}(i)-b^{-}(i)=\overline{y}_{iu}-\overline{y}_{zi}=\sum_{j\in V}y_{ji}-\sum_{j\in V}y_{ij}=\sum_{j\in V}\overline{y}_{ji}-\sum_{j\in V}\overline{y}_{ij}$$

from what we get

$$0=\sum_{j\in V}\overline{y}_{ji}-\sum_{j\in V}\overline{y}_{ij}-\overline{y}_{iu}+\overline{y}_{zi}=\sum_{j\in V\cup\{z\}}\overline{y}_{ji}-\sum_{j\in V\cup\{u\}}\overline{y}_{ij}=\sum_{j\in\overline{V}}\overline{y}_{ji}-\sum_{j\in\overline{V}}\overline{y}_{ij},$$

and hence

$$\sum_{j\in\overline{V}}\overline{y}_{ji}=\sum_{j\in\overline{V}}\overline{y}_{ij} \tag{21}$$

Conditions (21), (22) together with inequality $0 \le y_{ij}$ mean that the function $\bar{y}: \bar{E} \to R$ represents a flow in the network $\bar{G} = (\bar{V}, \bar{Y}, \bar{y})$ with the source z and the sink u. The magnitude of flow $\bar{y}$ is equal to any sum in (22).

Let positive numbers x_{ij} for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$ be residual debts after cyclic elimination. Then, similarly to (20), we can extend definition or the matrix $\{x_{ij}\}$ for z, u and for $i = 1, 2, \ldots, n$:

$$x_{zz} = x_{zu} = x_{uz} = x_{uu} = 0, \quad x_{zi} = b^-(i) \quad x_{iu} = b^+(i) \tag{23}$$

Condition (16) (the requirement that the balance of each firm remains unchanged) can be written in the form:

$$\sum_{j \in \bar{V}} x_{ij} = \sum_{j \in \bar{V}} x_{ji} \quad \text{for } i = 1, 2, \ldots, n \tag{24}$$

The relation (9) can be written as follows:

$$\sum_{i=1}^{n} x_{zi} = \sum_{i=1}^{n} x_{iu} \tag{25}$$

Finaly we add the above explained constraint

$$0 \le x_{ij} \le \bar{y}_{ij} \text{ for every } i \in \bar{V} \text{ and for every } j \in \bar{V}. \tag{26}$$

Conditions (24), (25), (26) conclude that $\mathbf{X} = \{x_{ij}\}_{i,j \in \bar{V}}$ is a flow in the network $\bar{G} = (\bar{V}, \bar{E}, \bar{y})$ where $\bar{y}: \bar{E} \to R$ is the edge capacity defined on $\bar{E}$. Moreover the flow magnitude of $\mathbf{X}$ is equal to (22). It is easy seen that (22) is also the magnitude of the maximum flow in network $\bar{G} = (\bar{V}, \bar{E}, \bar{y})$. So the problem of the total debt minimization can be formulated as the following graph theory problem: To find a maximum flow $\mathbf{X} = \{x_{ij}\}_{i,j \in \bar{V}}$ with minimum cost $f(X) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}$ in the network $\bar{G} = (\bar{V}, \bar{E}, \bar{y})$.

4. REVITALISATION

Cyclic debt elimination does not change balance of any firm. If we want to improve our algorithm to lower the total debt (14) we must add financial subsidiary to the system. This is sometimes called the revitalisation of the firm by payment of its payables.

Let us choose firm $i \in V$ with the negative balance $b(i)$. Then $b^+(i) = 0$ and $b^-(i) = -b(i)$. We assume that mutual debt compensation organiser provides an amout of $b^-(i)$ to the i-th firm. This enables paying its debts. Then, the total amount of mutual debts given by (14) can be decreased by the sequence of payments controlled by mutual debt organiser.

At first, let us investigate the problem of the total debt minimisation by paying all debts of the i-th firm. Let us construct the digraph $\overline{G}_i = (\overline{V}, \overline{E}, \overline{y}_i)$ where

$$\overline{y}_i(e) = \begin{cases} \overline{y}_i(e) & \text{if } e \in \overline{E},\ e \neq (i, u) \\ 0 & \text{if } e = (i, u) \end{cases} \tag{27}$$

To minimize the total debt by paying all debts of the firm i means to find a maximum flow $\mathbf{X} = \{x_{ij}\}_{i,j \in \overline{V}}$ with minimum cost in the network $\overline{G}_i = (\overline{V}, \overline{E}, \overline{y}_i)$. Assign $r(i)$ the difference between original total debt and total debt after the just described revitalisation procedure. We are looking for a firm revitalisation of which results in minimum total debt – i.e. for i with maximum $r(i)$. Other objectives can be used, too, for example maximization of ratio $\dfrac{r(i)}{b^-(i)}$.

Another approach is the following. We can put C money units into the system with objectives to minimize the total debt. This problems can be solved in digraph $\overline{\overline{G}} = (\overline{\overline{V}}, \overline{\overline{E}}, \overline{\overline{y}})$ where

$$\overline{\overline{V}} = \overline{V} \cup \{Z\},\quad \overline{\overline{E}} = \overline{E} \cup \{(Z, z)\},\ \text{and}\ \overline{\overline{y}}(e) = \begin{cases} \overline{y}(e) & \text{if } e \in \overline{E} \\ \sum_{i=1}^{n} b^-(i) - C & \text{if } e = (Z, z) \end{cases}.$$

The problem to minimize the total debt by putting C money units into system can be reduced to max–flow–min–cost problem in digraph $\overline{\overline{G}} = (\overline{\overline{V}}, \overline{\overline{E}}, \overline{\overline{y}})$.

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LOCAL LABOUR MARKETS CLASSIFICATION IN THE MORAVIA-SILESIAN REGION[1]

JANA HANČLOVÁ, JIŘÍ HORÁK, MILAN ŠIMEK

Abstract The empirical analysis is devoted to the local labour markets in the Moravia-Silesian region during period 1995-2002. The presented results document the possibilities how to use vice-dimensional classification methods for the local labour market evaluation using dynamic and spatial GIS statistics. The evaluation and classification of the local labour markets proceed via factor and cluster analyses. The stability of the vice-dimensional evaluation in a certain time period is investigated as well. The paper uses information provided by the departments of statistics of job centres in the Czech Republic.

1. INTRODUCTION

Traditional evaluation of the situation and development of the labour market just on the basis of the unemployment rate does not show well enough the different situation in different regions. The situation at the labour market should be described more comprehensively with regard to a number of factors affecting this situation both from the perspective of the labour supply and demand and from the perspective of the environment (among others also the geographical influences). The target of the paper is the evaluation of 302 selected local labour markets in municipalities of the Moravia-Silesian region in the period from 1995 to 2002 on the basis of 5 selected indicators from so called GIS statistics of job centres by means of extracted factors. The evaluation was done by means of hierarchical formation of 7 agglomerates.

The structure of the paper is adapted to the mentioned target. The first part informs the reader about the characteristics of the selected local labour markets by means of the monitored indicators from GIS statistics including the time development. Another part deals with the application of the factor analysis for extraction of latent factors which follow important features of the original set of indicators. The most important is the third part which classifies the mentioned local labour markets into 7 agglomerates on the basis of (un)similarity. The classification is dynamic

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[1] The paper is the outcome of the research grant n. 402/02/0855 "Labour market modelling using of geo-information technologies" funded by Grant Agency of the Czech Republic.

and spatial. The final evaluation summarises positive as well as negative results related to the proposed method of local labour markets classification and possibilities of further concentration of research or empirical activities.

The use of **multivariational methods** for spatial analyses consists above all in (Bailey, Gatrell 1995):

1) The reduction of quantity of data and research of multidimensional attributive space with the aim to indetify the small number of interesting subdimensions (or the combination of attributes) which can then be examined from the spatial point of view (the use of classic multivariational methods followed by the visualisation of results and their interpretation).
2) The research of spatial textures (patterns) and relations.
3) The spatial classification and discrimination.

The method of **minimum spanning tree** belongs to hierarchical agglomerative methods as well. The spanning tree consists of the system of lines (edges, according to the theory of graphs) among all observations which connect each observation (junction) with any other one and, at the same time, without any loops in the network (connected and acyclic graphs are the requirement of the theory of graphs on the structure of a tree type). The length of a line expresses unsimilarity of both connected observations. The minimum spanning tree is the tree of mentioned conditions with the minimum length. Bailey, Gatrell (1995) describe the spatial application of this method when they presume that the set of examined objects has got the spatial localisation and they recommend to represent the created graph (the minimum spanning tree) on the map to be able to prove the hypothesis that the observations which are close in the attributive space are also close in the geographical space and to indicate the respective deviations. However, the stated method is also used to define a part of the graph (for example, the street network) which is available within the defined limit (time, distance). Practical applications for this purpose have been described, for example, by You-Hong (1996).

2. THE CHARACTERISTICS OF THE ANALYSED TERRITORY AND DESCRIPTORS

The investigated area is the area of Moravia-Silesian region. Moravia-Silesian region includes the districts of Bruntal, Frydek-Mistek, Karvina, Novy Jicin, Opava and Ostrava-city. See Figure 1. This territorial unit is diversified by its nature and in the core there is Ostrava-Karvina agglomeration. It is a traditional industrial area with a high share of so called "heavy" industry, particularly coal and metallurgy industry, heavy engineering and chemistry. The engineering and pharmaceutical industries, electrotechnic, paper-mill, textile and food-processing industries are the other important branches of industry in the region.

Agriculture is developed in the whole area and the forestry can be found in the foot of Beskydy and Jeseniky mountains.

Since 1990 great restructuralisation has been running in the region together with making redundant thousands of employees. The region is one of the most affected in the republic. The typical feature is a fierce inhibition of the heavy industry and permanent increasing unemployment. The rate of unemployment was 17.6% until January 31, 2004.

Figure 1: Location of the Moravia - Silesian region in the Czech Republic

For the analysis the five characteristics have been chosen based on an expert estimation that influence the situation at local labour market. The indicators follows:

- rate of unemployment - indicator MN,
- share of unemployed between 15 - 24 years old on a total number of unemployed - indicator PC15-24_U,
- share of unemployed older than 50 years on a total number of unemployed - indicator PC50-99_U,
- share of unemployed with primary education level on a total number of unemployed - indicator PCVABC_U,
- share of long-term unemployed (more than 1 year) on total number of unemployed - indicator PCE_12U.

Rate of unemployment is one of the key indicators for the evaluation of the labour market troubles at macroeconomic, regional and local levels. Moravia-Silesian region belongs to the territories with the very high values of the indicator in the long-run. In the analysed period the indicator increased significantly.

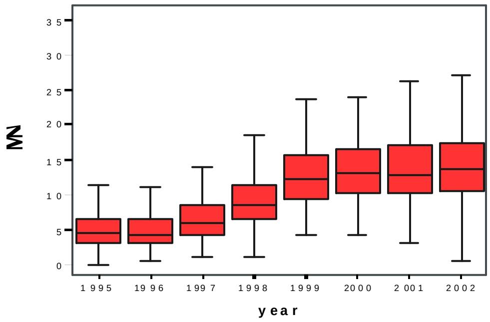

Figure 2: Unemployment rate in % in municipalities of the Moravia-Silesian region from 1995 to 2002

The share of job applicants 15-24 years old on total number of job applicants in %. This indicator expresses the share of the young proportion of the economic active population on total amount of the registered job applicants. A typical feature is a high differentiation of these values in municipalities in the long-run.

The share of job applicants older than 50 years on total number of job applicants in % was very stabile in the beginning of the analysed period. More significant increase for this group of job applicants has happened during 2000-2002. There are important differences within the region. For example, in Karvina district relatively favourable development could be influenced by a real social policy of the big firms aimed at dismissed people and by the accompanied social programmes.

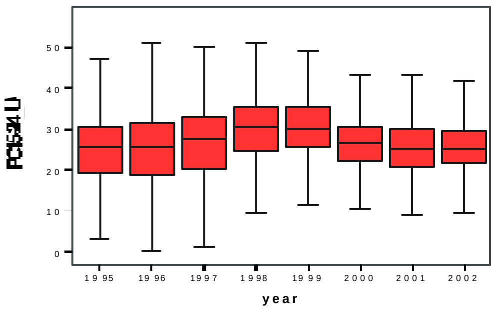

Figure 3: The share of job applicants 15-24 years old on total number of job applicants in % in municipalities of the Moravia-Silesian region from 1995 to 2002

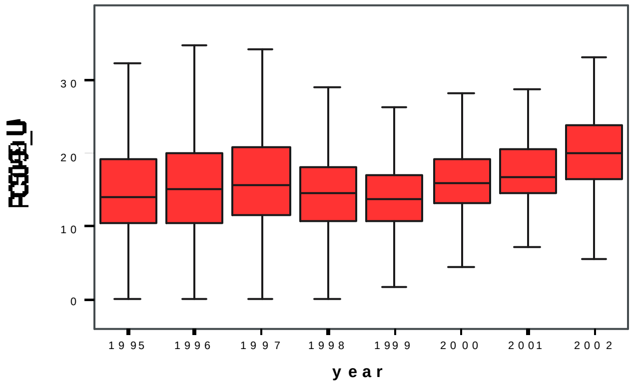

Figure 4: The share of job applicants older than 50 years on total number of job applicants in % in municipalities of the Moravia-Silesian region from 1995 to 2002

The share of job applicants who reached primary level education on total number of job applicants in % expresses the representation of non-qualified labour force in the territory. Job applicants who reached primary level education only represents the share of 30% on total amount of job applicants in the long-run. This indicator declined slightly in the analysed period.

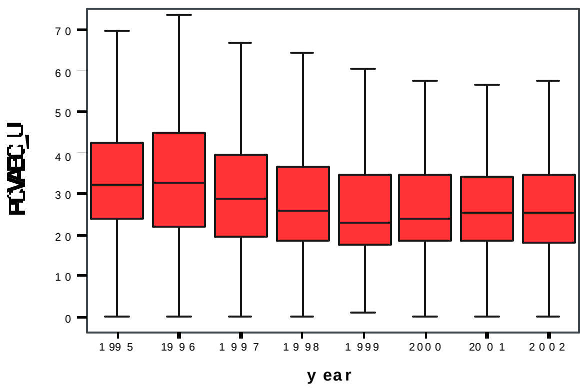

Figure 5: The share of job applicants who reached primary level education on total number of job applicants in % in municipalities of the Moravia-Silesian region from 1995 to 2002.

The share of job applicants registered more than 12 months on total number of job applicants in % reaches the value about 40% in 2002. The very significant increase of these values is apparent particularly during period 1999-2000. The high value indicates serious long-run troubles at labour market in the area.

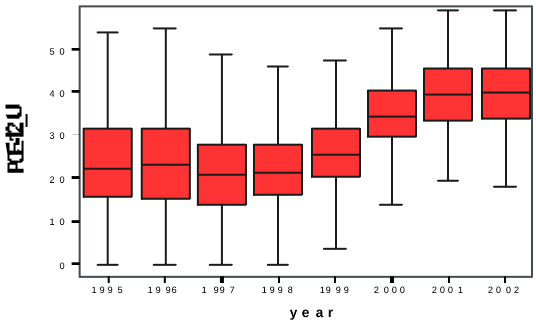

Figure 6: The share of job applicants registered more than 12 months on total number of job applicants in % in municipalities of the Moravia-Silesian region from 1995 to 2002.

3. FACTOR ANALYSIS OF THE SELECTED INDICATORS

Five basic indicators of the local labour markets have been described in the previous part of the paper. They have been calculated as an average of monthly values for each year of the analysed period. This chapter is devoted to the exploration indicator analysis. Based on its mutual correlation we put these determinants together to so called latent factors, that can be interpreted objectively and they express substantial features of the local labour markets. These extracted factors will classify the local labour markets in the following part.

The factor analysis had following phases:

- the estimation of correlation matrix,
- the estimation of factor load coefficients using the method of basic components
- the rotation of the factors for better interpretation using method VARIMAX,
- the estimation and using of the factor scores for setting up the profile of the local labour markets of municipalities.

Before starting the factor analysis the selected indicators were sorted according to the columns and per each year and individual municipalities were sorted according to the lines. Input data matrix of indicators included standardised data according to the individual indicators.

The result of the factor analysis for the whole observed period from 1995 to 2002 was the extraction of the two factors (see Table 1), which could be interpreted, and the proportion of the spread of indicators explained by these factors was at the satisfactory level 68%.

The factor F1 shows the problems of the *general global labour supply and demand* (i.e. the unemployment rate and the problems with a long-term unemployment). The factor F2 includes the problems on the side of the *labour supply structure* (high percentage of the young or elderly unemployed).

	Component	
	1	2
Zscore(MN)	,829	
Zscore(PCVABC)	,790	
Zscore(PCE12)	,727	
Zscore(PC50-99)		,911
Zscore(PC15-24)		–,670

Table 1: Rotated component matrix

4. LOCAL LABOUR MARKETS CLASSIFICATION IN 1995-2002

Local labour markets classification in the Moravia-Silesian region was based on the concept of homogeneity, i.e. searching for local labour markets with the high degree of mutual uniformity according to two extracted latent factors.

Again, the classification comes out from standardized values by means of the Z-score and the Pearson´s correlation coefficient was chosen as the similarity rate. The method of the hierarchical agglomeration using natural agglomerates centroids to search for agglomerates was chosen for the cross-sectional classification of local labour markets and during aggregation average distances of groups of couples of points among aggregates are evaluated. The main disadvantage of this classification method is marginalisation of space relations including the arrangement of hierarchical centres.

The following Figure 7 presents the development of seven clusters in 1995-2002. It is obvious, the figure and above mentioned process are based on the estimations of latent factors that could be interpreted objectively during the whole period. The generated clusters are formed in the individual years and they need not to be comparable for the local labour market according to the cluster value in time period.

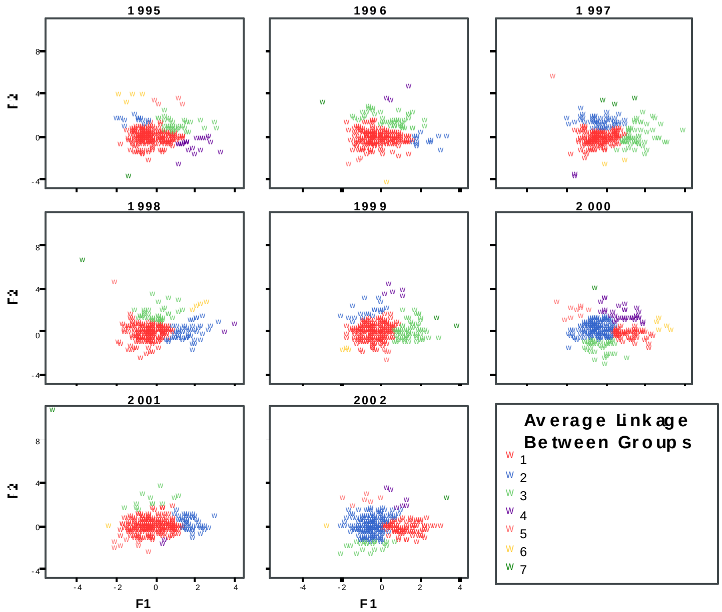

Figure 7 : The development of seven clusters in 1995-2002.

The following figure shows the results of the classification into 7 clusters for all municipalities in 1995.

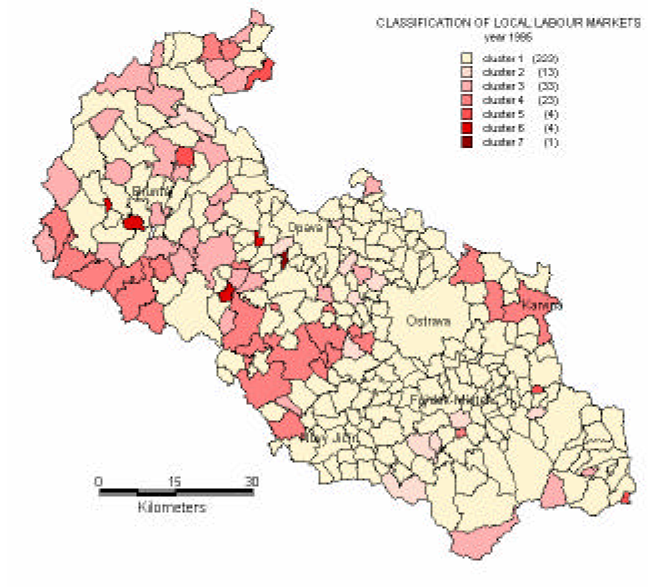

Figure 8: Classification of local labour markets in 1995

From the point of view of the allocation of the local labour markets to the clusters the received results for 1995 indicate several problem areas. These are, above all, peripheral areas of the region with the limited number of employers and bad accessibility from the point of view of the labour mobility, with the exception of the district of Karvina which suffered from the significant decline of mining industry.

5. CONCLUSION

The aim of this paper was the evaluation of 302 selected local labour markets of the Moravia-Silesian region in 1995-2002 using the indicators of job offices statistics.

The **factor analysis** resulted in 2 *extracted factors* expressing significant features of the original indicators. The factor F1 included global labour supply and demand problems. The factor F2 expressed problems on the side of the labour supply structure.

For the cross-sectional classification in individual years 1995-2002 the method of the hierarchical agglomeration into 7 levels was used. The results showed that the classification of the local labour markets is strongly sensitive to the location as well as changes of the extracted factors. The stability of the assigned classification level of a municipality in an eight-year examined period was evaluated as well.

Further results of this empirical study referred to significant problems of local labour markets at the level of municipalities with the small number of economic active inhabitants where there are significant changes of the analysed indicators. Marginalisation of further spatial relations among local labour markets affects the obtained results as well.

A number of spatial applications may require the created clusters to be spatially connected, i.e. clusters from geographically close objects were created. Traditional multivariational methods do not take note of this requirement. The easiest way is to classify the coordinates of objects into a set of observations as another two new variables, however, it is obvious that the optimalisation of the distance along both coordinate axes will to a certain extent go on independently (this may be solved by an introduction of an spatial indexation) and on the whole, of course, the coordinates are placed at the same level with other attributes and therefore it is not ensured that geographically homogeneous clusters will be formed. More advantageous possibility is the modification of the algorithm of the agglomerates formation with respect to spatial relations. For a lot of cases - e.g. administrative division of the area - we can require the areas joined into one cluster to neighbour. An offer to use a neighbourhood matrix is apparent. It contains information concerning neighbourhood among the individual pairs of areas. When using the agglomerate method for the creation of clusters we can join only those areas to the cluster that are neighbouring with another area already being involved in the cluster.

Further investigation will focus on the indicators including *labour mobility*. Labour markets will not be investigated in isolation.

We can sum up that the empirical study underlined the eligibility of the process for labour market classification and setting their rate of criticism in period 1995-2002. The acquired results and imperfections will be respected in future investigations.

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WASSERSTEIN METRICS AND EMPIRICAL DISTRIBUTIONS IN STABILITY OF STOCHASTIC PROGRAMS[1]

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Abstract Practical economic problems often ask for optimization procedures, not unfrequently with random inputs leading thus to stochastic programming models. The randomness is modelled through the underlying probability distribution, which is assumed to be completely known. But the "true" probability distribution is rarely fully known; instead, approximation or estimates are used. Some kind of stability of optimal values and optimal solution sets with respect to changes in the probability distribution is then required. This paper illustrates how a particular distance – Wasserstein metrics, measuring a distance between two probability distribution – affects the stability of stochastic optimization problems. Numerical examples for a choice of distributions and their empirical estimates are given.

1. INTRODUCTION

Stochastic programming specializes in problems where the uncertainty of input parameters has to be taken into account. Usually, the randomness is introduced to the model via probability distribution of the random variable. In such models, a full knowledge of the distribution is required. But in practice, its estimates and approximations have to be applied instead, due to modelling and numerical difficulties; for example, if the distribution for some input random parameter can be only estimated from a historical data series.

In a stochastic programming model, when replacing the original distribution with its approximation or estimate, one have to be cautious about resulting changes in optimal value and/or in optimal solution to the problem. We speaks about the *stability analysis* of stochastic programs *with respect to changes in* the underlying *probability measure.* Apart from other things, the stability analysis introduce another non-trivial task – to choose a suitable distance on the space of probability measures. A great attention has already been paid to this area in the literature, see e. g. [1], [2], [4], [5], [12], [13], etc. It turns out again that the problem is closely related to the properties of the original model and can not be treated separately from it.

1 This research was supported by the Grant Agency of the Czech Republic under Grant 402/01/0539.

Refering to recent results in the stability of stochastic programs (see e. g. [10]), we consider the Wasserstein metrics in the remainder of the paper — it is a convenient distance in many cases (see also Section 3). Let $\mathcal{P}(\Theta)$ be the space of all Borel probability measures on some Borel set $\Theta \subset R^s$ and denote

$$\mathcal{P}_1(\Theta) := \left\{ \upsilon \in \mathcal{P}(\Theta) : \int_\Theta \|\xi\| \upsilon(d\xi) < +\infty \right\}$$

the set of probability measures having finite first moment. Let $\mu, \upsilon \in \mathcal{P}_1(\Theta)$. The *1-Wasserstein metrics* is then defined by

$$W_1(\mu, \upsilon) := \inf_{\eta \in D(\mu, \upsilon)} \int_{\Theta \times \Theta} \|\xi - \bar{\xi}\| \eta(d\xi \times d\bar{\xi}),$$

where $D(\mu, \upsilon)$ is the set of all probability measures (of $\mathcal{P}(\Theta \times \Theta)$), for which μ and υ are marginal distributions. The practical dimension of the metrics appears when dealing with one-dimensional random variables; then the Wasserstein metrics reads (see [15])

$$W_1(\mu, \upsilon) = \int_{-\infty}^{+\infty} |F(t) - G(t)| dt \tag{1}$$

where F and G are distribution functions corresponding to the probability measures μ and υ. In the latter case, the metrics coincides with the Fortet-Mourier metrics, closely related to a more general concept of distances having ζ-structure. For details, we refer to the papers [1], [10], and the book [9]. The main disability of the Wasserstein distance is recognized while dealing with distributions having heavy tails (see [6]).

At the end of the paper, we consider the empirical distribution as the selected approximation method. The one-dimensional *empirical distribution function,* based on the sample of i. i. d. random variables $\xi_1, \xi_2, \ldots$ with common distribution function F, is defined by

$$F_n(z) = F_n(z, \omega) = \frac{1}{n} \sum_{i=1}^{n} I_{(-\infty; z]}(\xi_i(\omega)), \qquad z \in R \tag{2}$$

where I_A denotes the indicator function of the set A. It is well known that the sequence of empirical distribution functions converges almost surely to the distribution function F under rather general conditions.

2. PROBLEM FORMULATION

For $\mu \in \mathcal{P}(\Theta)$ (representing unknown original distribution) consider a general decision problem

$$\inf_{x \in R^n} \int_\Theta g(x; \xi) \mu(d\xi) \text{ subject to } \mu\{\xi \in \Theta : x \in X(\xi)\} \geq p_0 \tag{3}$$

where only general conditions on g and X are assumed: $G: R^n \times R^s \to R, X : R^S \Rightarrow R^n$ are both measurables, and $p_0 \in [0; 1]$ is a prescribed probability level. Recourse problems and problems with probabilistic constraints are special cases of (3). Denote $\varphi(\mu)$ the optimal value and $\psi(\mu)$ the optimal solution set to the problem (3).

In (3), we consider some estimate $\upsilon \in P(\Theta)$ (e. g. an empirical distribution defined by (2)) instead of μ. The next section recalls some Lipschitz and Hölder stability properties of $\varphi(\cdot)$ and $\psi(\cdot)$ with respect to the Wasserstein metrics. It deals particularly with the fixed constraint set $X \subset R^n$, i. e. with the recourse model of the form

$$\int_{\Theta} g(x;\xi)\,\mu(d\xi) \text{ subject to } x \in X \tag{4}$$

The stability results of Section 3 will next be applied when υ denotes an empirical distribution.

3. STABILITY RESULTS

Theorem1 Consider (4) where $\mu, \upsilon \in P_1(\Theta)$, X is compact, g is uniformly continuous on $R^n \times R^s$ and Lipschitz continuous in ξ for all $x \in X_0$ with a constant L independent on x. Then

$$|\varphi(\mu) - \varphi(\upsilon)| \le L W_1(\mu, \upsilon)$$

If in addition X is a convex set and $g(\cdot,\xi)$ is strongly convex function on X with parameter $\sigma > 0$, then

$$\|\psi(\mu) - \psi(\upsilon)\|^2 \le \frac{8}{\sigma} L W_1(\mu, \upsilon)$$

Proof See [3]; for a definition of strongly convex function see [11].

The strong convexity condition of g allows us to consider $\psi(\upsilon)$ as a (unique) point of R^n (however it is in fact a singleton).

Theorem 1 is a basic tool to estimate upper bound of difference in optimal value and optimal solution. First, upper bounds of Theorem 1 depend on the model structure by means of the Lipschitz constant L. Especially, the underlying structure can be complex so far as we can guarantee the Lipschitz continuity of g in its random component.

The other component of the upper bound given by Theorem 1 measures the distance between μ and ν by the Wasserstein metrics. We fix our attention to this point now. If the function $g(x,\xi)$ had a separable structure in random element, i. e. $g(x,\xi) = \sum g_i(x;\xi_i)$ where every ξ_i is one-dimensional random variable, then we could directly apply Theorem 1 on each of functions g_i. Here we take advantage of the fact that Wasserstein metrics is easily computable for one-dimensional distributions (see (1)). Consequently, upper bounds for changes in optimal value and optimal solution can be obtained.

4. STABILITY RESULTS

Let μ_n denote an empirical measure with distribution function F_n defined by (2). In [14], limiting distribution for $\sqrt{n}W_1(\mu, \mu_n)$ is calculated, where μ is the uniform distribution on (0, 1). Using the inverse transformation theorem, one could induce a limiting distribution of this statistics for arbitrary distribution; however, our aim is different. The inverse transformation theorem is not always applicable. The problem is that inverse distribution function F^{-1} could not always be given in its explicit form, and other approximation methods take place.

In the latter section, we give a short illustration on how some distributions deals with the Wasserstein metrics applied on themselves and theirs empirical versions. The goal is not to give an explicit formula for limiting distribution (this requires a more sophisticated theory groundwork, see again the book [14]), but to show how large errors one can look for when using different empirical distributions in practical optimization problems.

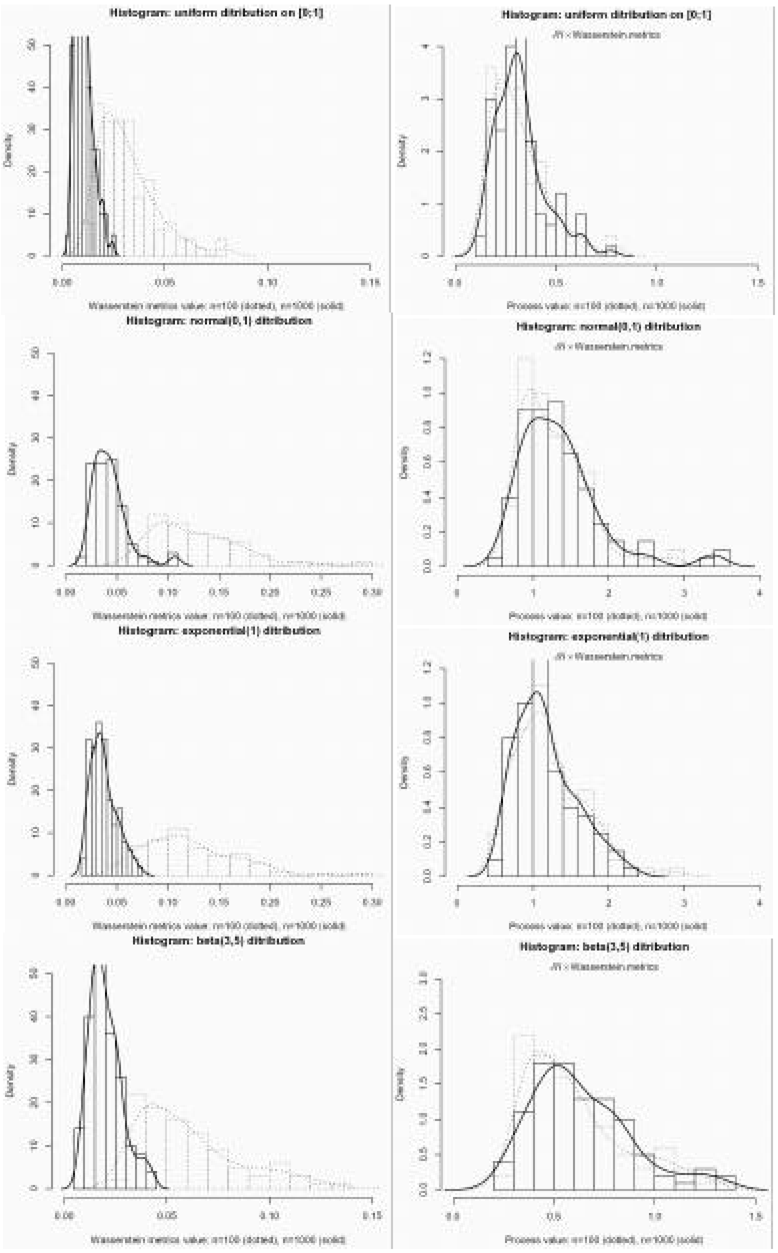
5. SIMULATIONS

We use the R programming language for calculations needed to this section. This free statistical tool is very flexible in implementing user-defined procedures. Nevertheless, we have chosen to use built-in but powerfull routines for integration and random number generation:

- (one-dimensional) integral estimate is based on QUADPACK numerical routines, see [8];
- pseudo-random number generator is of the type Mersenne-Twister: a twisted generalized feedback shift register generator, see [7]. Normal distribution is estimated using the inversion procedure and Wichura's algorithm AS 241 for quantile function, see [16].

The exact procedure for estimating $W_1(\mu, \mu_n)$ reads as follows: for a given distribution and a length n, a random sample is generated. Then the empirical distribution is set up and its absolute difference to the original distribution function is integrated giving an estimate for the Wasserstein distance. This procedure is repeated 100 times for each pair (distribution, n) in order to get basic statistical properties of estimates.

The first set of following histograms illustrates a fact that the Wasserstein distance between a distribution and its empirical estimate converges to zero as follows from well-known theoretical results about empirical distributions. The second set of histograms provides distributions of $\sqrt{n}W_1(\mu,\mu_n)$ (i. e. rate of the convergence). If one knows the Lipschitz constant of Theorem 1 (derived from the structure of the original model (4)), one can directly apply the theorem to obtain an estimate to the error arising when the original unknown distribution is replaced by its empirical estimate.



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ECONOMIC GROWTH MODEL AND POSSIBLE PROBLEMS OF THE SLOVAK ECONOMY

JAROSLAV HUSÁR

Motto
Jedine hlboká a dokonalá znalost **makroekonómie** poucuje o spôsobe, ako funguje ekonomický systém, ako v nom tecú toky príjmov a výdavkov, o tom, ci je hospodárstvo zdravé, a ci je jeho chod vratký. Len ona vysvetluje, z ktorého subsystému (casti) pochádza choroba (neduh), pre aké príciny a ako sa rozmáha. Ona ukazuje cestu inštrumentom (liekom). Skrátka povedané, len zato, že ju vieme (ovládame) sa stávajú zjavnými ostatné jednotlivosti, takže ak nepozná niekto túto hlavnú cast ekonómie, nie je prospešné poznat ostatné.

INTRODUCTION

Recently our economy vent through mass adjustments. The policy makers were persuaded that to restore macroenomic stability by bringing the budget close to balance and pursuing tight monetary and credit policies is the prime economic interest. Our new economy started by privatizing government-owned firms by selling them. The next step was to liberalize prices by removing price controls and allowing markets to begin operating. We have liberalized foreign trade. Little care was devoted to economic growth as the main source of economic welfare. In this paper we concentrate on analysis of some aspects of economic growth that are ahead of the Slovak economy.

1. ECONOMIC PERFORMANCE OF THE SLOVAK ECONOMY

To understand growth and differences in income levels among countries, we need to understand what determines the growth of factor of production and technical knowledge. The performance of the economy must be seen through many economic indicators. Despite progress made in reducing spending as a share of GDP since 1998 and in cutting tax rates from high levels, the large deficit – estimated at 6,5% of GDP (on ESA 95 basis) in 2001 – and the significant rise in public debt raise serious concerns. The Slovak economy is emerging slowly from a period of sluggish growth, that is the prerequisite for the welfare, which is expected by people. The

Maastricht' criteria require to meet fixed values of chosen economic indicators. But to evaluate the performance, we have to look into the structure of GDP components (table 1). In order to have a deep understanding the economic problems of the Slovak economy, we picked up data on the GDP structure for Slovakia, USA and GB.

The fundamental challenge facing Slovakia, as we can see from the data, is the increase in the consumption expenditure share. There are no small differences but high one. We could put the question for whom the economic system of Slovakia is producing. Share of the C in the GDP is only cca. 0,50. In USA and GB this share reache the value of more than 0,65. The next surprising ratio is the share of investment in GDP. For the period of existence of Slovak Republic this share is floating round the value of 0,35. A high rate of capital formation is not reflected in the standard of living or in the labour productivity.

Table 1: The share of the GDP components in SR, USA and GB

GDP components	SR		USA		GB	
	bil. Sk	v %	bil. USD	v %	bil. GBP	v %
Consumption expenditures, C	322,3	**49,3**	4390,0	**68,9**	382,7	**64,2**
Investment expenditures, I	252,7	**38,6**	875,2	**13,7**	92,9	**15,6**
Inventories, I	-20,6	-0,03	16,8	0,002	-1,9	-0,003
Government expenditures G	146,2	22,3	1157,1	18,1	132,4	22,2
exports, X	368,8	56,4	660,1	10,4	139,8	23,4
imports, M	415,5	63,5	725,8	11,4	149,1	25,0
GDP	653,9	100	6374,0	100	596,1	100

So the efficiency of investment as well as efficient capital allocation are essential to raising standard of living and labour productivity. After the need to raise productivity growth, next major problem is the sluggish short-term performance of the economy (growth) and the inadequate job creation.

The next important insight in economic performance is the structure of consumption expenditures. The consumption expenditure is not the homogenius item. It is usual to divide it to three components: expenditures on durable goods (C1), on non-durable goods (C2) and expenditures on the services (C3). This more detailed view of the US economy is seen in the Table 2. It is ans advanced economy so we can have an idea about the possible path of economic system and possible objectives of economy in transition.

Table 2: The structure of selected indicators for US economy

Year	GDP, bil. USD	*C*	*I*	*C/HDP*	*I/GDP*	*C1/C*	*C2/C*	*C3/C*
1995	7400,5	4969,0	1143,8	0,671	0,154	0,12	0,30	0,58
1996	7813,2	5237,5	1242,7	0,670	0,159	0,12	0,30	0,58
1997	8300,8	5524,4	1383,7	0,665	**0,167**	0,12	0,29	0,59
1998	8759,9	5848,6	1531,2	**0,667**	0,175	0,12	0,29	0,59
1999	9248,4	6254,9	1625,6	0,676	0,176	0,12	0,29	0,59

The reader can see the importance of the expenditures on services in USA as the component of overall expenditures of households. This comparison suggest the ground for economic policy in the Slovak economy.

To speak about required changes in our economy asks for increasing the growth rate. Investment spending is a central topic in macroeconomic policy for two (more) reasons. First, fluctuations in investment account for much of the movement of GDP in the business cycle. Second, investment spending determines the rate at which the economy adds to its stock of physical capital, and thus helps *determine the economy's long-run growth and productivity performance.* This was the reason for construction the model and on his base to analyze the Slovak economy prospects. We want to rely on gross and net investment. To have the idea of numerical relationship, look at the gross and net investment as a percentage of GDP in USA:

Table 3: Gross and net investment (per cent of GDP)

	1960-80	1980-90
Gross investment	18,1	17,6
Net investment	606	5,5

Source: OECD, Historical Statistics, 1992

As we can see, *net investment declined* in the second period *more than gross investment*. We can only make a hypothesis what was the reason. But now we have an idea about the real relationship in the field of investment in an advanced economy. As it is known the rates of depreciation depends on the type of capital. For example, the useful life of structures is decades whereas that of office equipment is only a few years.

Now look at some numerical values in the Slovak economy. We had some problems to get the same data. But in any case we succeeded to get data that are in Table 4:

Table 4: Gross fixed capital formation and its consumption:

Indicator	1996	1998
GDP	100	100
Gross fixed cap. formation	0,37	0,36
Consumption of capital	0,17	0,19

As we see the gross investment as the component of GDP in Slovakia is very high. Perhaps it should suggest the other capital allocation to get a higher growth rate.

2. AGGREGATED GROWTH MODEL OF SLOVAK ECONOMY WITH POSSIBLE ECONOMIC PROBLEMS

To study and analyze the growth problems of Slovak economy we want through growth models. The simplest growth models stem from the analysis of capital accumulation in the absence of capital accumulation. The basic model, generally known by the label Harrod – Domar, is constructed in a form that is both the simplest and most inflexible[1]. The variables in the Harrod-Domar model in continues terms are output (income) Y and labor force L, both as flows per unit of time, together with the capital stock K at any time and its derivative ∂K as the flow of investment. All variables are taken as continuous and differentiable functions of time. Hence the basic model has three variables: Y, K, and L; and their paths are determined by three equilibrium conditions:

Full capacity	$K = vY$	
Investment = saving	$\partial K = sY$	(1)
Full employment	$L = uY = L_0 e^{nt}$	

In this model it is required thata the growth rate of L must be the rate of growth (g) of Y. But the given rate of L is n, natural rate. It can be shown that $g = \frac{s}{v} = n$. If this essential condition is satisfied, then steady-state growth paths of the three variables are:

$$Y = Y_0 e^{gt}, \quad K = K_0 e^{gt} \quad \text{and} \quad L = L_0 e^{gt}.$$

To speak about the economic growth of Slovak economy using only verbal techniques will not help to find the solution. Critical problem, as we have seen in tables, is the ratio of investment and GDP. From this stems our interest to formulate a model that is focusing on macroeconomic variables I, K and Y. On the base of the model (1) we constructed a model that consists of these equations:

$$I_t = J_t - U_y \quad (2)$$

$$K_t = vY_t \quad (3)$$

$$J_t = sY_t \quad (4)$$

$$I_t = K_{t+1} - K_t \quad (5)$$

where

I_t – net investment
J_t – gross investment
K_t – stock of capital
U_t – depreciation
Y_t – GDP

1 Pozri E. Domar, Essays in the Theory of Economic Growth, New York, 1957.

As we see there are three crucial parameters: v, s and μ. The economic meaning attached to v is that desired capital stock is a constant multiple (v) of output. Our equation (3) is the core equation, starting point for the model development. If this equation is valid, then this statement is valid also:

$$K_{t+1} = vY_{t+1} \tag{6}$$

For the next development of the model we have said that the depreciation is constant ratio of the capital, that is that U_t is μ ratio of the capital stock, i.e.:

$$U_t = \mu K_t = \mu \cdot v \cdot Y_t \tag{7}$$

Now manipulating wit the expressions from (2) to (7) we can deduce the analytical form of the relationship Y_t as a function of time, t. First of all from (5) we see that:

$$K_{t+1} - K_t = J_t - U_y.$$

In this expression all the variables can be changed to Y_t (substituted) that requires these three relations (3), (4) a (7). The solution is

$$v \cdot (Y_{t+1} - Y_t) = s \cdot (Y_t - \mu \cdot v \cdot Y_t).$$

Rearranging this equation (expression) after several steps we finally get this model:

$$Y_{t+1} = \left(1 + \frac{s}{v} - \mu\right)Y_t \tag{8}$$

The next steps in this model are clear enough in general terms. Let the variable $t = 0, 1, 2, \ldots, n$, we will get:

$$Y_1 = \left(1 + \frac{s}{v} - \mu\right)Y_0,$$

$$Y_2 = \left(1 + \frac{s}{v} - \mu\right)Y_1$$

$$\ldots$$

$$Y_n = \left(1 + \frac{s}{v} - \mu\right)Y_{n-1}$$

After the required substitutions (they can be done by reader) we finally come to this result:

$$Y_n = \left(1 + \frac{s}{v} - \mu\right)^n Y_0 \tag{9}$$

To be more familiar this equation, we can say that $\rho = \left(\frac{s}{v}\right) - \mu$. We managed to get new form of (9) that can be written in this form (solution of difference equation):

$$Y_n = (1 + \rho)^n Y_0 \tag{10}$$

This particular formulation says that the parameter ρ may be regarded to be a rate of growth. From the equation (9) the reader can se that all the parameters of the original equations *s*, *v*, and μ are important in the growth rate formation. They have their own economic meaning. But as it is seen we need to know the initial value that is related to all parameters and a variable Y_0. Parameters may be estimated. We now have a good model (instrument) that can serve for making economic analysis. This was the main purpose of this model formulation.

Now we can turn to the economic analysis and problems that can be expected in Slovak economy from the point of view of real relationships in macroeconomic variables. From equation (3) we see that desired capital stock is a fixed multiple of the level of income. The fixed multiple (*v*) is the ratio of the desired capital to output (*v* = *K/Y*), called capital-output ratio. It mainly depends on the fixed capital of the firms. In the business sector, we are assuming that firms have no choice – they must have capital on hand in an amount equal to (*v*) times the level of output. But the growth rate heavily depends on this value. From the equation (4) we can see the importance of the parameter *s*. It can be read off from the GDP components. If it is high, as in Slovakia, it means that we need not only financial resources for investment but we need other resources too: plants, equipment, labour force, new technologies, new professions, etc. This creates enormous problems to every economy. But the high level of *I* in the GDP in the same time means that the *C* had to go down. No less important is the parameter μ.

In our paper we want to present some calculations concerning the Slovak economy. First of all, we made a choice of the value of the parameters that way as to get for each three values the growth rate to be 7%. Just to show you some results, in Table 4 you can see the particular values.

In variant A we fixed the depreciation on the level that was observed in the Slovak economy and we also fixed the value of *s*, the ratio of gross investment in DGP on the level that is a little bit more than real (0,39).

Table 4: The parameters of the model

alternative	*s*	*v*	μ	ρ
A	0,40	2	0,13	0,07
B	0,60	3	0,13	0,07
C	0,28	2	0,07	0,07

Growth rate in this case is 7%. In the variant B we fixed again the rate of depreciation and we accepted the capital – output ratio that can be fined in modern (developed) economies (3). This requires the *s* to be equal to 0,6 (very high) if we want to have a growth rate equal to 7%. It means that consumption expenditures in Slovakia (and other expenditures) must suffer, household sector may be unsatisfied. Economic development takes place when the economic welfare of a country's people increases over a long period. In the third alternative we made a courageous assumption that the depreciation will be on very low level – 7% and that capital output ratio reaches the value 2. So for all the three alternatives the growth rate is 7%. All these alternatives say that the initial stages of reform before entering the EU are very difficult. Table 4 shows estimates of GDP/capita in coming 24 years based on the model (10) for Slovakia in comparison with the same data for Austria. But it must be said that we took as an initial value for the GDP/capita to be 8560 USD. This number is the result of accepting the so called PPP recalculations. Next that must be said, we made an assumption that the growth rate of the Austrian economy will follow the rate of 2,2 per cent, experienced in last years.

Table 5: The GDP growth for SR and Austria

Year	GDP	GDP/cap.,USD	Austria
0	997,00	8560,00	26643,50
1	1066,79	9159,20	27229,66
2	1141,47	9800,34	27828,71
3	1221,37	10486,37	28440,94
4	1306,86	11220,41	29066,64
5	1398,34	12005,84	29706,11
6	1496,23	12846,25	30359,64
7	1600,96	13745,49	31027,55
8	1713,03	14707,67	31710,16
9	1832,94	15737,21	32407,78
10	1961,25	16838,82	33120,76
11	2098,54	18017,53	33849,41
12	2245,44	19278,76	34594,10
13	2402,62	20628,27	35355,17
14	2570,80	22072,25	36132,98
15	2750,75	23617,31	36927,91
16	2943,31	25270,52	37740,32
17	3149,34	27039,46	38570,61
18	3369,79	28932,22	39419,16
19	3605,68	30957,48	40286,38
20	3858,08	33124,50	41172,69
21	4128,14	35443,21	42078,48
22	4417,11	37924,24	43004,21
23	4726,31	40578,94	43950,30
24	5057,15	**43419,46**	**44917,21**

The Table 5 shows that while the level of per capita income in Austria is very high, our level of per capita income is relatively low (see also the OECD Economic

Survey). If Slovak real income continues to grow at 7% and Austrian at 2,2%, Slovakia will catch up to the Austria (using dollars) in 24 – 25 years. This may sound im pos si ble, but it is close to what Ja pan achieved: be tween 1950 and 1990, Ja pan's in come (mea sured by ICP method – in ter na tional com par i sons pro ject) rose rel a tive to that of USA from 18 to 78 per cent of US per ca pita GDP. Be fore Slovakia stands a method to fig ure out how to reach a growth rate of 7 per cent, we have to con cen trate on real sources of Slo vak econ omy.The model shows what de ter mines the growth rate of out put over a long pe ri ods, at least we think that the picked up pa ram e ters are of a big im por tance.

CONCLUSION

In this pa per we have shown the pos si ble anal y sis of the eco nomic growth us ing model as a method to come to nu mer i cal re sults. First we con structed a model that relies upon econ0omic relationship between principal macroeconomic vari ables: gross in vest ment, net in vest ment, cap i tal and out put. It was shown that all three investigated strategies are very troublesome. Out idea was to concentrate on the 7percent growth rate, that is very high in the given and per spec tive con di tions of our econ omy. It seems that Solow's sur pris ing con clu sion that says that over 80 per cent of the growth in out put per la bour hour over the stud ied pe riod was due to tech ni cal prog ress will also be valid in the next 20 – 30 years. In dis cuss ing some prob lems of the Slo vak econ omy, we draw on the an a lytic and em pir i cal in sights of growth rate that could be an ob jec tive for Slovakia.

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EXCHANGE RATE VOLATILITY AND MONEY MARKET

IVANIČOVÁ ZLATICA, RUBLÍKOVÁ EVA

After changing the fixed exchange regime to the flexible one, the exchange rate of the Slovak crown has a tendency to degrease. The Slovak central bank, because of the negative current account, endeavor to retain the level of exchange rate. The authorities of the monetary policy regular practice the exchange rate intervention focused on the weakness of the Slovak crown.

We focused our analysis on the exchange rate movements. In general it is possible to mentioned two approaches going from the same proposals. Both approaches use the simple relation

$$\dot{e} = f\left(I,\ y,\ e\right) \tag{1}$$

where $\dot{e}$ is log of the realized rate of change in the exchange rate, I is the log of the weighted average of price of the domestic good, y is the log of the output level and e is log of the exchange rate. If the output level y is assumed to be exogenous, we can speak about Dornbusch model, if a price level is rigid, we have Mundell-Fleming model.

1. FORMULATION OF THE MODEL

Money demand can be specified in log linear form

$$m_D = I_C + \alpha y - \beta r \tag{2}$$

where m_D is the log of the demand of the money stock, I_C is the log of the consumption price index and r is log of the domestic interest rate. The consumption price index I_C ca be defined as a weighted average of the price level of the domestic good (I) and the price level of the foreign good expressed in domestic currency ($e + I_F$),

$$I_C = \sigma I + \left(1-\sigma\right)\left(e + I_F\right) \tag{3}$$

In the freely flexible exchange rate system considered here, the supply of money (m_S) can be controlled by the domestic monetary authorities. Money market equilibrium then implies that

$$m_S = I_C + \alpha y - \beta r \tag{4}$$

Assuming open interest rate parity yields

$$r = r_F + \pi \tag{5}$$

where we define π as the expected rate of depreciation of the domestic currency, and r_F as the foreign interest rate.

In the framework of a deterministic model, rational expectations imply perfect foresight. Put differently, in this model there is no uncertainty about the parameters nor about the future exogenous variables. One can, therefore, write that

$$\pi = \dot{e} \tag{6}$$

(technically $\dot{e}$ is the derivative of the log of the exchange rate with respect to time), i.e. the expected rate of change in the exchange rate must equal the realized rate of change.

Combining the previous equations (3) and (6) into one equation which describe the financial equilibrium (money market equilibrium and interest parity)

$$r = r_F + \dot{e}$$

$$m_S = \sigma I + (1-\sigma)(e + I_F) + \alpha y - \beta(r_F + \dot{e})$$

$$\beta\dot{e} = \sigma I + (1-\sigma)e + (1-\sigma)I_F + \alpha y - \beta f_F - m_S$$

and rearrange:

$$\dot{e} = \frac{\sigma}{\beta} I + \left(\frac{1-\sigma}{\beta}\right) e + \frac{1}{\beta} z \tag{7}$$

where

$$z = \alpha y + (1-\sigma) I_F - \beta r_F - m_S$$

or

$$\dot{e} = \frac{\alpha}{\beta} y + \left(\frac{1-\sigma}{\beta}\right) e + \frac{1}{\beta} z' \tag{8}$$

where

$$z' = \alpha I + (1-\sigma) I_F - \beta r_F - m_S \ .$$

The equation (7) is a first order differential equation, which describes the motion of the exchange rate in a perfect foresight environment – Dornbusch approach to analysis of variability of exchange rate.

The equation (8) is a first order differential equation describes the motion of the exchange rate in a perfect foresight environment (the dynamics in the money market) – Mundell-Fleming approach to analysis of variability of exchange rate.

Equilibrium is obtained when $\dot{e} = 0$, i.e. from equation (7) the acquired equation is

$$I = -\frac{1-\sigma}{\sigma} e - \frac{1}{\sigma} z \tag{9}$$

from equation (8) the acquired equation is

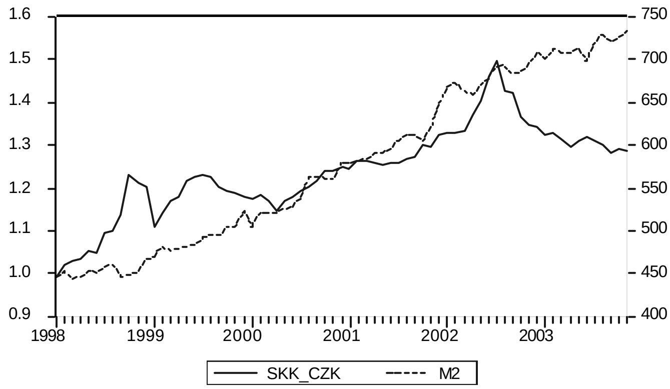
$$y = -\frac{1-\sigma}{\sigma} e - \frac{1}{\alpha} z' \ . \tag{10}$$

Relation (10) is nothing as a LM curve. Thus, along the LM curve the exchange rate is at rest because the money market is in equilibrium.

In this presentation we will analyze equitation (7) based on the simplified version of the Dornbusch model. In particular it does not make a distinction between traded and non-traded

2. ANALYSIS OF THE CHANGE OF THE EXCHANGE RATE SKK/CZK

In the papers [5],[6],[7] and [8] exchange rate SKK/CZK was analyzed on the basis of purchasing power parity and interest rate parity and development of the inflation rate. Some relation is possible survey also between evaluation of money in foreign currency (exchange rate) and money stock. This relation is depicted on the picture 1.



Picture 1: Development the exchange rate SKK/CZK and Money stock M2

In 1998 Slovak Republic adopted floating mode of exchange rate, what carried unstableness of exhcange rate and exchange rate was overestimated concerning on slow growth of money stock. During the period of years 2000 and 2001 till the March 2002 there is the growth of money stock and at the same time also the growth of exhange rate, so depreciation of Slovak crown was gradual to the Czech one. On July 2002 this development has changed. Although money stock (M2) in Slovakia is increasing, so there are expectation of increasing the exchange rate and the depreciation of the currency, there is possible to see that the Slovak crown appreciate. It is the consequence of the reversal development of the cross exchange rate of USD and EUR. Because of EUR appreciate and the value of USD is decreasing, Slovak crown also appreciate together with the appreciation of EUR not only to the USD but also to the Czech crown as well. Slovak crown has stable development to the EUR, even it is up to standard of Masstricht criteria concerning of stability of currency.

That inforamtion carried us to the idea to test validity of simple Dornsbuch model for development of exchange rate. The analysis was done on the basis of monthly data from January 1998 till November 2003 for exchange rate SKK/CZK, inflation rate and money stock M2. The formula (7) was modified. For estimation of parameters the inflation rate was used instead of consumer price index and money stock M2 instead of the variable z as well.

The results are following:

$$d\log SKK\_CZK_t = -0{,}262 - 0{,}014\log INF_t - 0{,}187\log SKK\_CZK_{t-1} + 0{,}042\log M2_t$$

(0,17) (0,00) (0,06) (0,03)

$$D - W = 1{,}76.$$

Because M2 is not statisticaly significant at 10 percent level of significance, it was excluded from the model and the new estimation was done as:

$$d\log SKK\_CZK_t = -0{,}008 - 0{,}013\log INF_t - 0{,}114\log SKK\_CZK_{t-1}$$

(0,11) (0,00) (0,03)

$$D - W = 1{,}83.$$

On the basis of gaining results we can state that because of floating exchange rate and strongly open Slovak economy the change of money stock is weakly rebuting in the movement of exchange rate. Essential effect on the development of the exchange rate has the price level, which is strongly depending on the values of the foreign exchange rates, namely USD and EUR.

Development of the money stock M2 and the exchange rate SKK/CZK in the period from January 1998 till November 2003 was estimated by means of Box–Jenkins methodology with the following models:

Model for M2:

$$\left(1-\phi_1 B^{12}\right)\left(1-B\right)M2_t = K + a_t$$

or

$$\left(1-0{,}552B^{12}\right)\left(1-B\right)M2_t = 1{,}668 + a_t$$

(0,11)

Model for SKK_CZK:

$$\left(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3-\phi_4 B^4\right)\left(1-B\right)SKK\_CZK_t = a_t$$

or

$$\left(1-0{,}165B-0{,}216B^2-0{,}069B^3+0{,}325B^4\right)\left(1-B\right)SKK\_CZK_t = a_t$$

(0,12) (0,12) (0,12) (0,12)

Forecasts for money stock M2 and exchange rates SKK/CZK for period December 2003 – November 2004 are listed in the table 1.

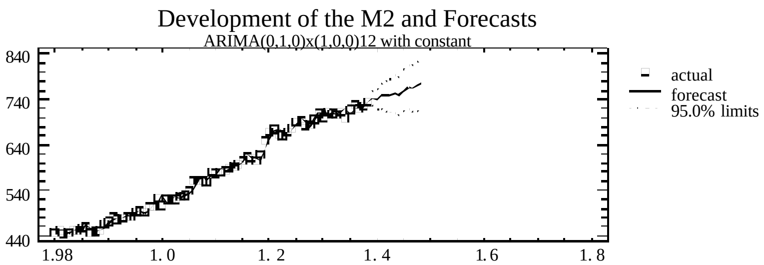
Table 1

Period	Money stock M2[1/]			Exchange rate SKK/CZK[2/]		
	Forecast	Lover 95,0%	Upper 95,0%	Forecast	Lover 95,0%	Upper 95,0%
		Limit			Limit	
Dec. 2003	742,456	726,287	758,626	1,234	1,188	1,281
Jan. 2004	740,038	717.17	726,906	1,227	1,155	1,298
Febr. 2004	747,835	719,828	775,842	1,225	1,128	1,322
March 2004	747.459	715,119	779,799	1,232	1,111	1,353
April 2004	748,74	712,583	784,897	1,236	1,100	1,371
May 2004	753,445	713,837	793,053	1,240	1,092	1,388
Jun 2004	747,051	704,269	789,832	1,243	1,086	1,400
July 2004	759,928	714,192	805,663	1,242	1,078	1,407
Aug. 2004	766,841	718,331	815,351	1,242	1,070	1,414
Sept. 2004	764,312	713,178	815,446	1,240	1,061	1,419
Oct. 2004	769,293	715,663	822,922	1,239	1,053	1,423
Nov. 2004	774,605	718,59	830,619	1,239	1,045	1,432

Source: own calculation

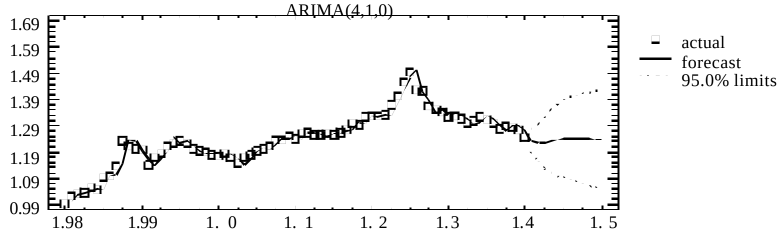
1/ forecast is done from December 2003 to November 2004 because of shorter time series

2/ forecast is done from February 2004 to January 2005



Picture 2

On the picture 2 is depicted the development of the Money stock (M2) in the period from January 1998 to November 2003 and the forecast from December 2003 to November 2004. It is possible to observe, that the value of the money stock will slowly growth. The exchange rate SKK/CZK has a tendency to be rigid, see picture 3.



Picture3

Anal y sis of in fla tion was pub lished in [8]. The fore cast of M2, ex change rate and in fla tion is pre pared for fur ther anal y sis within the frame of the VEGA Pro ject

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APPLICATION OF ALTERNATIVE METHODS IN RECALCULATION OF THE HUMAN DEVELOPMENT INDEX[1]

JOSEF JABLONSKÝ

Abstract The Human Development Index (HDI) is an aggregate indicator that measures the average achievements in different countries in three basic dimensions of human development - life expectancy, education and standard of living measured by GDP per capita. Its calculation is multiple criteria decision-making (MCDM) problem and currently the basic utility function approach with the identical weights for all the indicators is used. The paper compares published results with results given by several other MCDM techniques. Moreover, the HDI can be estimated by data envelopment analysis (DEA) models. Two basic DEA models for HDI calculation are formulated and the results given by them are presented.

1. HUMAN DEVELOPMENT INDEX

The Human Development Index (HDI) is an indicator that is published for all the UN member countries every year by United Nations Development Programme within the Human Development Report. The complete version of this report can be found and downloaded from the www page hdr.undp.org. The HDI measures and compares the quality of human life in different world countries. The calculation of the HDI is based on the following four criteria:

- life expectancy at birth in years (LE),
- adult literacy rate in % (LR),
- combined primary, secondary and tertiary gross enrolment ratio in % - (the number of children enrolled in each level of schooling divided by the number of children in the age group corresponding to that level (GER),
- GDP per capita in US$.

The following three indices are calculated form these criteria:

1. *Life expectancy index* (LEI) is calculated according to the following formula:

$$\frac{\textit{life experiency}\ \text{in the given country} - 25}{85 - 25}.$$

[1] The paper is supported by the Grant Agency of Czech Republic - grant no. 402/03/1360 and corresponds to the research program of the Faculty of Informatics and Statistics no. CEZ:J18/98:311401001.

The LIE is from 0 to 1. The highest value was reached by Japan (0,94) with the life expectancy 81,3 years in 2002. In the contrary, the lowest value is 0,14 for Zambia. The LIE for the Czech Republic is 0,83 (life expectancy 75,1 years).

2. *Education index* (EI) consists of two criteria - adult literacy rate and gross enrolment ratio. It is calculated as the weighted average with weight 2 for literacy rate and weight 1 for gross enrolment ratio. The highest values are greater than 0,99 (highly developed countries as Norway, Sweden, Great Britain, etc.). The lowest value is 0,17 for Niger. The EI for the Czech Republic is 0,91.
3. *GDP index* is calculated as follows:

$$\frac{\log(\mathrm{GDP}) - \log(100)}{\log(40000) - \log(100)}$$

The highest GDP has Luxemburg (53780 US$ per capita). Thus, the value of the GDP index computed by the above formula is greater than 1. For the HDI calculation purposes is this value lowered to 1,00. The lowest GDP index has Sierra Leone 0,26 (GDP is only 470 US$ per capita). The Czech Republic has GDP 14720 US$ and reaches the GDP index 0,83.

The HDI is calculated as the average of the three mentioned indices. The summary statistics and results for selected countries are given in Table 2.

2. MULTICRITERIA ANALYSIS OF THE HDI

The calculation of the HDI is a general multiple criteria decision making problem with the aim to receive complete ranking of all the alternatives (countries) by four criteria that have to be maximized. This problem can be solved by one or several available MCDM methods.

Criterion	**Weight**	**Indifference threshold**	**Preference threshold**
life expectancy	1/3	1	5
literacy rate	2/9	1	5
gross enrolment ratio	1/9	4	20
GDP	1/3	2	10

Table 1: Parameters used in PROMETHEE class methods

The method currently used for calculation of the HDI is the simplest utility function approach. The weights used for our further analysis are given in Table 1 - they correspond to the weights used in the standard approach. The values of the HDI were recalculated by two frequently used MCDM methods - TOPSIS and

PROMETHEE. TOPSIS is based on minimisation of the distance of alternatives form both the ideal and basal alternatives. Except weights of the criteria this method does not need any further parameters. PROMETHEE class methods need specification of the type of so called generalised function that is used in comparison of pairs of alternatives and calculation of intensity of preference between them. In our experiments we used linear generalised function with indifferent area. Parameters of this function for the criteria are presented in Table 1.

Country	LE	LR	GER	GDP	HDI	Rank	PR II	Rank	TOPSIS	Rank
Norway	78,7	99,0	98	29,62	0,944	1	0,812	1	0,753	5
Iceland	79,6	99,0	91	29,99	0,942	2	0,808	2	0,761	4
Sweden	79,9	99,0	100	24,18	0,941	3	0,801	3	0,629	18
Australia	79,0	99,0	100	25,37	0,939	4	0,796	4	0,656	12
Netherlands	78,2	99,0	99	27,19	0,938	5	0,794	6	0,697	8
Belgium	78,5	99,0	100	25,52	0,938	6	0,791	7	0,659	11
USA	76,9	99,0	94	34,32	0,937	7	0,790	8	0,862	2
Canada	79,2	99,0	94	27,13	0,937	8	0,796	5	0,695	9
Japan	81,3	99,0	83	25,13	0,932	9	0,783	9	0,650	14
Switzerland	79,0	99,0	88	28,10	0,931	10	0,782	10	0,717	7
Denmark	76,4	99,0	98	29,00	0,930	11	0,761	16	0,737	6
Ireland	76,7	99,0	91	32,41	0,930	12	0,769	14	0,817	3
UK	77,9	99,0	100	24,16	0,930	13	0,774	12	0,628	19
Finland	77,8	99,0	100	24,43	0,930	14	0,773	13	0,634	17
Luxemburg	78,1	99,0	73	53,78	0,929	15	0,760	17	0,961	1
:										
Czech Rep.	75,1	99,0	76	14,72	0,861	32	0,532	31	0,420	38
Slovak Rep.	73,3	99,0	73	11,96	0,836	39	0,403	35	0,364	43
:										
Burundi	40,4	49,2	31	0,69	0,337	171	-0,709	170	0,080	166
Mali	48,4	26,4	29	0,81	0,337	172	-0,693	168	0,069	172
Burkin Faso	45,8	24,8	22	1,12	0,330	173	-0,717	173	0,059	174
Niger	45,6	16,5	17	0,89	0,292	174	-0,728	174	0,053	175
Sierra Leone	34,5	36,0	51	0,47	0,275	175	-0,744	175	0,062	173

Table 2: HDI recalculation by MCDM methods.

The results of the multicriteria analysis were given by our original add-in application in MS Excel environment for solving MCDM problems called Sanna. The comparison of country rankings given by the PROMETHEE II method with the ranking according to the published HDI values show they very close relation. In the contrary, the ranking given by the TOPSIS method is not so correlated with the HDI rankings. In several cases the ranking differs very significantly - e.g. Guinea Equatorial has the HDI equal to 0,664 (rank 116), the similar result is given by the PROMETHEE II method (110) but the TOPSIS method assigns rank 39 (it can be explained by very high value of GDP in this country). The results for the first 15 countries, Czech and Slovak Republic and the last 5 countries show Table 2. The first four columns of this table contain the raw data, i.e. criterion values according to our four criteria. The next three pairs of columns contain the HDI value, net flow given by the PROMETHEE method and utility value computed by the TOPSIS method always with the rankings of the countries by these indicators.

3 RECALCULATION OF THE HDI BY DEA MODELS

Data envelopment analysis (DEA) is a tool for measuring the relative efficiency and comparison of general decision making units (DMU). The DMUs are usually characterised by several inputs that are spent for production of several outputs. The DEA models try to measure the efficiency of transformation of inputs into outputs and assign to the DMUs the efficiency score that can be used for ranking of inefficient units (efficient ones have the efficient score equal to 1). The basic DEA models can be used for recalculation the HDI. We propose the following two DEA model:

1. The model that considers four outputs (LE, LR, GER and GDP) and one input that has identical unit values for all the countries (DMUs). We suppose the output-oriented model with constant return to scale with one uncontrolled input (the input values can not be changed). The model produces the efficiency score that can be taken as the estimation of the HDI computed in the standard way. The mathematical formulation of this models can be written as follows:

$$
\begin{aligned}
\text{maximize} \quad & z = \phi - \varepsilon \sum_{i=1}^{r} s_i^+ \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j = x_{iq}, && i = 1, 2, ..., m, \qquad (1)\\
& \sum_{j=1}^{n} y_{ij}\lambda_j - s_i^+ = \phi y_{iq}, && i = 1, 2, ..., r,\\
& \lambda_j \geq 0, s_i^+ \geq 0, s_i^- \geq 0
\end{aligned}
$$

where z is the efficiency score of the unit DMU $_q$, m is the number of inputs, r is the number of outputs, n is the number of DMUs, x_{ij}, i = 1, 2, ..., m, j = 1, 2, ..., n, is the value of the input i for the DMU$_j$, y_{ij}, i = 1, 2, ..., r, j = 1, 2, ..., n, is the value of the output i for the DMU$_j$, s_i^+, i = 1, 2, ..., r are slack variables corresponding to outputs and l_j and f are decision variables of the model. This formulation of the model does not make it possible to restrict the weights of the inputs and outputs. The weights restriction can be useful in order to include all the criteria in the model by a given minimum way. That is why it can be useful to work with dual model:

$$\begin{aligned} &\text{minimize} && z=\sum_{i=1}^{m} v_i x_{iq} \\ &\text{subject to} && \sum_{i=1}^{m} v_i x_{ij} - \sum_{i=1}^{r} u_i y_{ij} \ge 0, \qquad j = 1, 2, ..., n, \\ &&& \sum_{i=1}^{r} u_i y_{iq} = 1, \\ &&& u_i \ge d_i, v_i - \text{free}, \end{aligned} \tag{2}$$

where u_i and v_i are the weights of the i-th input and i-th output respectively and decision variables of the model at the same time, and d_i is the lower bound for weight of the i-th output. The units with efficiency score z = 1 are efficient, the efficiency score z > 1 indicates inefficiency (for purposes of comparison with the HDI we will work with the reciprocal values of efficiency scores 1/ z).

2. The second model takes into account one input (GDP) and three outputs (LE, LR and GER). This models tries to measure the efficiency of transformation of the country wealth (measured by GDP) into outputs influencing in the positive way the human life in the country. We suppose the output-oriented model with variable return to scale with weights restrictions. Here, the model (2) changes in the following way:

$$\begin{aligned} &\text{minimize} && z=\sum_{i=1}^{m} v_i x_{iq} + \varphi \\ &\text{subject to} && \sum_{i=1}^{m} v_i x_{ij} - \sum_{i=1}^{r} u_i y_{ij} + \varphi \ge 0, \qquad j = 1, 2, ..., n, \\ &&& \sum_{i=1}^{r} u_i y_{iq} = 1, \\ &&& u_i \ge d_i, v_i \ge e_i, \end{aligned} \tag{3}$$

where e_i is the lower bound for weight of the i-th input.

Country	HDI rank	DEA (1) d=0,001	Rank	DEA (1) d=0,003	Rank	DEA (2) d=0,001	Rank	DEA (2) d=0,003	Rank
Norway	1	1,000	1	1,000	1	0,990	40	0,970	41
Iceland	2	1,000	1	0,984	10	0,985	53	0,954	65
Sweden	3	1,000	1	0,995	5	1,000	1	0,997	15
Australia	4	1,000	1	0,996	3	0,998	23	0,989	23
Netherlands	5	0,999	8	0,995	4	0,992	35	0,977	36
Belgium	6	1,000	1	0,995	6	0,997	26	0,987	27
USA	7	1,000	1	1,000	1	0,979	65	0,940	82
Canada	8	0,995	13	0,983	12	0,990	41	0,968	43
Japan	9	0,998	9	0,956	19	0,996	27	0,954	64
Switzerland	10	0,989	17	0,968	16	0,982	58	0,948	73
Denmark	11	0,997	12	0,991	7	0,988	46	0,964	51
Ireland	12	0,994	14	0,984	11	0,978	69	0,936	85
UK	13	0,998	11	0,989	9	0,998	22	0,989	22
Finland	14	0,998	10	0,990	8	0,998	25	0,988	24
Luxemburg	15	1,000	1	0,977	13	0,955	88	0,877	117
:									
Czech Rep.	32	0,991	15	0,972	15	1,000	1	1,000	1
Slovak Rep.	39	0,954	36	0,874	42	0,981	62	0,945	77
:									
Burundi	171	0,486	172	0,461	171	0,763	156	0,759	148
Mali	172	0,558	164	0,490	167	0,786	152	0,741	155
Burkin Faso	173	0,527	168	0,462	170	0,642	171	0,574	172
Niger	174	0,518	169	0,446	175	0,701	164	0,636	167
Sierra Leone	175	0,494	171	0,464	169	0,993	33	0,955	63

Table 3: HDI recalculation by DEA models.

The results of the model (2) can be directly compared with the published HDI values. Table 3 contains rankings of the countries according to the original HDI values in the first column. In the next two columns there are the efficiency scores computed by the model (2) with all the weights for the outputs greater than 0,001. Under these conditions seven of the countries are identified as efficient. In order to classify these countries it is possible to increase the weights of the outputs and to compute the efficiency scores again. Increasing the weights leads to the lower (or equal) efficiency scores of the DMUs. In this case some of the DMUs efficient in the model with lower bounds become inefficient in the model with higher bounds. In this way it is possible to classify the DMUs. The results for lower bounds for the outputs 0,003 are presented in the next two columns of Table 3. Here we can see that the most efficient countries according to the model (2) are Norway and USA.

The results of the second DEA approach, i.e. model (3), are different comparing to the previous ones. If we consider the lower bounds for the outputs and the input 0,001 there are efficient 17 countries, among them several developing countries as Tanzania, Guyana, Georgia, etc. Many of the highly developed countries are not efficient according to this model but it is necessary to remark that they are very close to the efficient frontier and their efficiency scores are close to 100%. The less efficient among all the countries are Angola and Mozambique with the efficiency score slightly greater than 50%. The last four columns of Table 3 contain the efficiency scores and ranking of selected countries with lower bounds for weights of the outputs and the input 0,001 and 0,003 respectively.

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SERVICE SYSTEM DESIGN IN THE PUBLIC AND PRIVATE SECTORS

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1. INTRODUCTION

Needs and requirements of human society or particular social groups form various demands, which are usually spread over a geographical area. An effective satisfaction of the demands is possible only if the corresponding service provider concentrates its sources at several places of the served area and if he provides the service in these places only or if he serves the demands at their positions by trips starting and terminating at these places. To denote source of a service, we shall use the term "facility", which is mend to include a broad set of entities such as factories, warehouses, transportation terminals, schools, hospitals, day-care centres, public administration offices, emergency warning sirens and others. An addressee of the service will be denoted by the term "customer", even if he has hardly anything common with this term in the market sense in many cases.

Within frame of this paper we restrict ourselves on such problems, in which a finite number of customers and finite number of possible facility locations are considered, what could be pretty good approximation of most of real cases. The great deal of a service system design will be done, when question on a number of facilities and their locations are answered. We shall distinguish two classes of service systems, which differ in objectives. The first considered class is formed by so-called public systems and the second one is referred as private systems. When a system from the first class is designed, the objective is stated as minimization of a social cost subject to service each customer or, in addition, some customer's equity in access to the service may be demanded. On the contrary to a public systems design, a private system designer accents profit maximization or capture of larger market share. In this case, service of some disadvantageous demands can be omitted.

In the following sections we formulate several typical models of both private and public system designs and we shall discuss means of corresponding solving approaches. Then we show a way, how to rearrange all reported models to general one, problem instances of which are solvable by exact algorithms for considerable large size.

2. PUBLIC SERVICE SYSTEMS

As preliminaries for model construction, we introduce the following notation of particular terms, which will be used throughout the whole paper. Let J denote a finite set of customers and if a quantity of customer's demand can be expressed by a real number, then demand of customer $j \in J$ be denoted by b_j. Let I denote a finite set of possible facility locations and let d_{ij} denote the distance between location $i \in I$ and the location of customer $j \in J$. The decision on facility location at place $i \in I$ is modelled by zero-one variable $y_i \in \{0,1\}$, which takes value 1 if a facility should be located at i and it takes value 0 otherwise.

2.1 Maximum Distance Model

In this problem a customer's demand is covered if its distance from some located facility is less or equal than given constant D. This case corresponds to problems like emergency warning sirens locations or health centre location. The objective is to cover all the customer demands with minimum number of located facilities. The classical approach [Current, 2002], [Marianov, 2002] introduces set $N_j=\{i \in I: d_{ij} \le D\}$ of possible locations, from which demand of customer j can be satisfied. Then the model of the corresponding set covering problem can be established in the form:

$$\text{Minimize} \sum_{i \in I} y_i \tag{1}$$

Subject to

$$\sum_{i \in N_j} y_i \ge 1 \qquad \text{for } j \in J. \tag{2}$$

2.2 p-Centre Problem

This problem consists in minimizing the maximum distance between customer and the nearest located facility, when pre-determined number of p facilities is given. This problem arises, when limited number of fire-stations or first-aid stations should be located so that time, in which the worst situated customer can be served, be as small as possible. To form a model of this problem, auxiliary variables $z_{ij} \in \{0,1\}$ for each $i \in I$ and $j \in J$ are introduced to assign customer j to possible location i. Furthermore, we add an another variable $w \ge 0$, which is used as upper bound of the distances between customer j and assigned location i. The p-center problem can be formulated as follows:

$$\text{Minimize } w \tag{3}$$

Subject to

$$\sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \tag{4}$$

$$z_{ij} \leq y_i \qquad \text{for } i \in I \text{ and } j \in J \tag{5}$$

$$\sum_{i \in I} y_i = p \tag{6}$$

$$\sum_{i \in I} d_{ij} z_{ij} \leq w \qquad \text{for } j \in J. \tag{7}$$

2.3 p-Median Problem

A problem of this type arises, when the number of facilities is fixed and average distance or average weighted distance between customer and the nearest located facility should be minimized. In the public sector, one might want to locate, for example, public administration bureaus in such a way as to minimize the total distance that citizens must traverse to reach their closest bureau. If J denotes set of dwelling places in the area, b_j number of inhabitants at $j \in J$ and if we use the previously introduced variables, an associated model to this problem can be established this way:

$$\text{Minimize} \sum_{i \in I} \sum_{j \in J} b_j d_{ij} z_{ij} \tag{8}$$

Subject to (4), (5), (6).

3. PRIVATE SERVICE SYSTEMS

Making use of above introduced notation, we try to formulate models of two broadly spread problems in the private sector.

3.1 Maximum Covering Location Problem

Similarly to the previous models, the number of facilities is fixed at value p, but, on the contrary to public the systems, not each customer must be served. We consider that service of customer j brings profit, which is proportional to its demand b_j. The customer is considered to be served if there is located at least one facility within distance D from the customer. Making use of sets N_j introduced in sub-section 2.1 and introducing auxiliary variables $x_j \in \{0,1\}$ taking value 1, if customer j is served, we can established the following model:

$$\text{Maximize} \sum_{j \in J} b_j x_j \tag{9}$$

Subject to

$$\sum_{j \in N_j} y_i \geq x_j \qquad \text{for } j \in J \tag{10}$$

$$\sum_{i \in I} y_i = p. \tag{11}$$

3.2 Fixed Charge Location Problem

We consider a simpler version of the problem stated in [Current, 2002] or [Kubanová, 1997]. The number of facilities in not fixed here, but each facility location at $i \in I$ is connected with fixed charge f_i, which doesn't depend on demand quantity satisfied from this location. Furthermore, costs c_{ij} are introduced to express cost connected with servicing customer j from a facility located at i. The objective is to satisfy all customer demands and to minimize the total costs including both fixed charges and service costs. This problem often emerges, when a distribution system is designed. Making use of previously introduced variables y_i and z_{ij}, the model can be stated as follows:

$$\text{Minimize} \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \tag{12}$$

Subject to (4), (5).

4. SOLVING METHODS

There are various exact methods for solving the particular problems mentioned above [Buzna, 2003], [Current, 2002], [Erlenkotter, 1978] [Marianov, 2002],. Besides, having formulated linear mathematical programming model, some of general solvers based on branch and bound approach can be used [Jablonský, 2002]. As shown in [Chocholác ek, 1998] for particular problem, special approaches win from the time consumption point of view, if special and general solver approaches are compared. Neverless, if the studied problem is modified e.g. by addition of a new constraint or by slight change in the objective function, then these special approaches are useless.

That is why we focus on the possibility, how to rearrange the broad spectrum of problems to general one, for which a smart solving tool has been developed. The general problem has form of the above fixed charge location problem with limited number of used facilities. The model has the following form:

$$\text{Minimize} \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \tag{13}$$

Subject to

$$\sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \tag{14}$$

$$z_{ij} \le y_i \qquad \text{for } i \in I \text{ and } j \in J \tag{15}$$

$$\sum_{i \in I} y_i \le p \tag{16}$$

The associated problem can be solved by the approach reported in [Janácek, 2000], where Lagrangean multiplier f is introduced for constraint (16), which is to be relaxed. Then the problem (14)-(16) can be reformulated this way: Find $f \geq 0$, so that values of variables y_i, $i \in I$ of the optimal solution of problem (17), (14), (15) meet constraint *(16)* as equality. The considered objective function is

$$\sum_{i\in I}(f + f_i)y_i + \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} \tag{17}$$

If f is fixed, then problem (17), (14), (15) forms an uncapacitated location problem. To solve the problem for nonnegative values of f and $\{c_{ij}\}$, procedure *BBDual* [Janácek, 1997] was devised and implemented. Being tested during computational experiments with large networks, the procedure proved to be able to solve large size problems quickly enough to be used repeatedly in more complicated algorithms.

To find demanded value f, an algorithm was completed [Janácek, 2000], in which function $Q(f, \mathbf{c})$ gives number of variables y_i which value is equal to one in the optimal solution of problem *(17), (14), (15)* for given $f, \mathbf{c}$.

0. **Set** $f_{\min} = 0,\ f_{\max} = \sum_{j\in J} \max\{c_{ij},\ i \in I\},\ f = \dfrac{(f_{\max} - f_{\min})}{2}$.
1. **While** $(Q;(f, \mathbf{c}) \neq p)$ and $(f_{\max} - f_{\min} \geq \varepsilon)$ **repeat**

   If $Q(f, \mathbf{c}) > p$ **then set** $f_{\min} = f$, **otherwise set** $f_{\max} = f$.

   Set $f = \dfrac{(f_{\max} - f_{\min})}{2}$.

It is necessary to remark that the optimal solution $\langle y, \mathbf{z}\rangle$ of problem (17), (14), (15) for resulting f need not necessarily meet constraint (16) as equality [Janácek, 2000].

5. PROBLEM REARRANGEMENTS

In this section, there will be shown that it is possible to rearrange each of the above models to the form of model (13)-(16). The reformulation is not necessary for the fixed charge location problem (3.2). In that case, it is sufficient to omit the constraint, which bounds the number of used facilities. Furthermore, the p-median problem (2.3) with nonnegative coefficients can be considered as a form of the generalized problem for $f_i = 0$.

The rearrangement of models 2.1, 2.2 and 3.1 can be done by the following way:

Maximum Distance Model (2.1) can be reformulated to model (13)-(16) defining $f_i = 1$ for $i \in I$ and $c_{ij} = 0$ for $j \in J$ and $i \in N_j$ and $c_{ij} = 2$ otherwise.

p-Centre Problem (2.2) can not be transformed directly to model (13)-(16), but such a model can be derived that its solution provides approximate solution of the p-centre problem and by repeating the solving process, the resulting solution can be made arbitrary precise. To reach this goal, lower and upper bounds on optimal value of w must be determined. Let us denote the current bounds $w_{\min}$ and $w_{\max}$.

The interval $[w_{\min}, w_{\max}]$ is divided into r equidistant parts by values $w_1 < w_2 < \ldots < w_r = w_{\max}$. Then the surrogate costs c_{ij} for $j \in J$ and $i \in I$ are established in accordance to the rule: $c_{ij} = 0$ for $d_{ij} < w_1$; $c_{ij} = (|J| - p)^k$ for $w_k \le d_{ij} < w_{k+1}$ for $k = 1, \ldots, r-1$; $c_{ij} = (|J| - p)^r$ for $w_r \le d_{ij}$. With this costs p-median problem can be solved and its largest value c_{ij}, for which optimal $z_{ij} = 1$ determines $\underline{k}$ and associated $w_{\underline{k}}, w_{\underline{k}+1}$ form new lower and upper bounds on the original problem.

Maximum Covering Location Problem (3.1)

To rearrange the former model, we introduce assignment variables $z_{ij} \in \{0, 1\}$ taking value *1* if and only if customer *j* is assigned to place $i \in I$. Then we can rewrite the former model as:

$$\text{Maximize} \sum_{j \in J} \sum_{i \in N_i} b_j z_{ij} \tag{18}$$

Subject to

$$\sum \sum_{i \in N_i} z_{ij} \le 1 \qquad \text{for } j \in J \tag{19}$$

$$z_{ij} \le y_i \qquad \text{for } j \in J \text{ and } i \in N_j \tag{20}$$

$$\sum_{i \in I} y_i = p \tag{21}$$

Further we add one "fictive" place i_0 to each neighbourhood N_j obtaining new neighbourhoods $\underline{N}_j = N_j \cup \{i_0\}$. Now we introduce slack variables z_{i0j} for each constraint (19) and defining $\underline{c}_{ij} = b_j$ for each $j \in J$ and $i \in N_j$ and $\underline{c}_{i0j} = 0$ for each $j \in J$, we obtain model, in which constrains (19) take form of equality.

After this arrangement, constant

$$|J| \cdot \underline{C} = |J| \cdot \max\{\underline{c}_{ij} : j \in J, I \in \underline{N}_j\} = \underline{C} \cdot \sum_{j \in J} \sum_{i \in \underline{N}_i} z_{ij}$$

can be subtracted fromthe (18) without loss of generality. This way, a new objective function with non-positive coefficients $(\underline{c}_{ij} - \underline{C})$ is obtained and when maximization is replaced with minimization of the objective function with nonnegative coefficients $c_{ij} = \underline{C} - \underline{c}_{ij}$, than the only difference between the general model and the obtained one consists in summation over sets $\underline{N}_j$. This can be adjusted by introducing some prohibitive constant $C > \underline{C}$ and coefficients $c_{ij} = C$ for $j \in J$ and $i \notin \underline{N}_j$ together with the associated variables z_{ij} .

6. CONCLUSIONS

We have shown that a broad spectrum of location problems originating in both private and public sectors can be rearranged to the form of uncapacitated location problem with simple constraint on number of located facilities. Furthermore, an approximative approach based on uncapacitated location technique was referred, which very often reaches an optimal solution. An advantage of the suggested solving process consists in speed, with which it is possible to obtain an exact solution of a very large problem in comparison with general integer programming solvers. Results reported in this contribution accents importance of further development of exact solving techniques for uncapacitated location problem on various types of underlying networks. Unfortunately, our preliminary experience indicates that if capacitated constraints are considered in a location problem, its intractability increases and disables exact solving of large instances. Overcoming of this obstacle may represent a topic of a further research.

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ACKNOWLEDGEMENT

This work has been supported by research project VEGA 1/0498/03.

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MULTIOBJECTIVE PROGRAMS AND MARKOWITZ MODEL

VLASTA KAŇKOVÁ

Abstract. It happens rather often that multicriteria optimization programs with objective functions depending on a probability measure correspond very often to economic situations. A "classical problem" of optimal portfolio selection belongs to this class of optimization problems; the operator of mathematical expectation appears there in objective functions. Employing the approach for general type of one and multiobjective stochastic programming problems we analyse the stability (considered w.r.t. the probability measures space) and behaviour of empirical estimates in the above mentioned problem.

Keywords: Portfolio selection, Markowitz approach, multiobjective stochastic programming problems, efficient solution, strongly efficient solution, stability, empirical estimates.

1. INTRODUCTION

A very simple "underlying portfolio problem" (considered w.r.t. one period) is known as the following one. To choose, under the assumptions of no transactions costs, no taxis and under the assumption that short sales are not permitted, among n different infinitely divisible financial assets (see e.g. [1], [2]). Evidently (under the above mentioned assumptions) we obtain the following simple optimization problem. Find

$$\max \sum_{k=1}^{n} \xi_k x_k$$

subject to

$$\sum_{k=1}^{n} x_k \le 1, \qquad x_k \ge 0, \quad k = 1, \ldots, n \tag{1}$$

where x_k denotes the fraction of the unit wealth invested in the asset k, ξ_k denotes the return of the asset $k = 1, 2, \ldots, n$ at the end of the considered period.

If ξ_k, $k = 1, \ldots, n$ are known constants, (1) is simple linear programming problem. However, in real–life situations ξ_k, $k = 1, \ldots, n$ are random variables. If we denote

$$\mu_k = E_F \xi_k, \quad c_{k,j} = E_F(\xi_k - \mu_k)(\xi_j - \mu_j), \xi_k, k = 1, \ldots, n,$$

then it is reasonable (under a well–known philosophy) to set to the portfolio selection two–objective optimization problem.

Find

$$\max \sum_{k=1}^{n} \mu_k x_k, \qquad \min \sum_{k=1}^{n} \sum_{j=1}^{n} x_k c_{k,j} x_j$$

subject to

$$\sum_{k=1}^{n} x_k \le 1, \quad x_k \ge 0, \quad k = 1, \ldots, n \tag{2}$$

E_F denotes the operator of mathematical expectation corresponding to the distribution function F of the random vector $\xi = (\xi_1, \ldots, \xi_n)$

Evidently, there exists only rarely a possibility to find an optimal solution simultaneously with respect to the both criteria. Markowitz suggested (see e.g. [2]) to replace the problem (2) by one–criterion optimization problem of the form.

Find

$$\max \left(\sum_{k=1}^{n} \mu_k x_k - m \sum_{k=1}^{n} \sum_{j=1}^{n} x_k c_{k,j} x_j \right) \text{subject to } \sum_{k=1}^{n} x_k \le 1, \quad x_k \ge 0, \quad k = 1, \ldots, n \tag{3}$$

where $m \ge 0$ is a constant. Obviously for every $m \ge 0$ there exists $\lambda \in \langle 0,1 \rangle$ such that the problem (3) is equivalent to the following one.

Find

$$\max \left\{ \lambda \sum_{k=1}^{n} \mu_k x_k - (1-\lambda) \sum_{k=1}^{n} \sum_{j=1}^{n} x_k c_{k,j} x_j \right\}$$

subject to

$$\sum_{k=1}^{n} x_k \le 1, \quad x_k \ge 0, \quad k = 1, \ldots, n \tag{4}$$

The problems (2), (3) and (4) depend on the distribution function F Since mostly F is not exactly known, a stability investigation (considered with respect to a probability measures space) as well as an investigation of statistical estimates are very serious problems connected with the original problem. Of course, it means to investigate the stability and statistical estimates of the (properly) efficient points set in the case of the problem (2) (see e.g. [9], [12]). The stability investigation of the multiobjective stochastic optimization problems has been started in [4].

In this contribution we shall deal with the stability and empirical estimates in the case of the problem (4) or equivalently in the problem (3). To this end we employ the results known from one–criteria stochastic programming problems (see [7], [8],

[11]). Furthermore we shall modify the results achieved for multiobjective stochastic programming problems [9], [12] to apply them to two–objective problem (2). To this end we assume.

A.1 There exist finite

$$\mu_k = E_F \xi_k, \quad c_{k,j} = E_F (\xi_k - \mu_k)(\xi_j - \mu_j), \quad k, j = 1, \ldots, n \tag{5}$$

2. SOME DEFINITIONS AND AUXILIARY ASSERTIONS

To investigate the stability and empirical estimates concerning the problems (2) and (4) (consequently (3)) we recall some definitions and prove some auxiliary assertions.

1. One-Objective Approach

Definition 1.

Let $\bar{h}$ be a real–valued function defined on a convex set $K \subset R^n$. $\bar{h}$ is a strongly concave function with a parameter $\bar{\rho} > 0$ if

$$\bar{h}(\lambda x^1 + (1-\lambda)x^2) \geq \lambda \bar{h}(x^1) + (1-\lambda)\bar{h}(x^2) + \lambda(1-\lambda)\bar{\rho}\|x^1 - x^2\|^2$$

for every $x^1, x^2 \in K$ $\lambda \in \langle 0, 1 \rangle$ ($\|\cdot\|$ denotes the Euclidean norm in R^n).

Proposition 1. [6]

Let $K \subset R^n$ be a nonempty, compact, convex set. Let, moreover, $\bar{h}$ be a strongly concave with a parameter $\bar{\rho} > 0$, continuous, real–valued function defined on K. If $x^0 = arg \max_{x \in K} \bar{h}(x)$, then

$$\|x - x^0\|^2 \leq \frac{2}{\bar{\rho}} |\bar{h}(x) - \bar{h}(x^0)| \text{ for every } x \in K.$$

Lemma 1.

Let $K \subset R^n$ be a nonempty set. Let, moreover, $h_i, i = 1, \ldots, r$ be Lipschitz functions on K with the Lipschitz constants $\bar{L}_i$. Then $h_{\bar{\lambda}}$, $\bar{\lambda} = (\bar{\lambda}_1, \ldots, \bar{\lambda}_r), \bar{\lambda}_i \geq 0$, $i = 1, \ldots, r$, $\sum_{i=1}^{r} \bar{\lambda}_i = 1$ defined by

$$h_{\bar{\lambda}}(x) = \sum_{i=1}^{r} \bar{\lambda}_i h_i(x), \quad x \in K$$

is a Lipschitz function on K with a Lipschitz constant not greater then $\sum_{i=1}^{r} \bar{L}_i$.

To introduce the next auxiliary assertion we define the function g_λ on $R^n \times R^n$, the set X and the matrix $\boldsymbol{C}$ by

$$g_i^*(x, z) = \sum_{i=1}^{n} z_i x_i, \quad g_2^*(x, z) = -\sum_{k=1}^{n}\sum_{j=1}^{n} x_k (z_k - \mu_k)(z_j - \mu_j) x_j, \quad z = (z_1, \ldots, z_n)$$

$$X = \left\{ x \in R^n, \; x = (x_1, \ldots, x_n) : \sum_{k=1}^{n} x_k \le 1, \quad x_k \ge 0, \; k = 1, \ldots, n \right\}$$

$$g_\lambda(x, z) = \lambda g_1^*(x, z) + (1-\lambda) g_2^*(x, z), \quad \lambda \in \langle 0, 1 \rangle, x = (x_1, \ldots, x_n),$$

$$\mathbf{C} = (c_{i,j}), \text{ where } \mu_j, c_{i,j}, \; i, j = 1, \ldots, n \text{ are defined by the relation (5).} \qquad (6)$$

Lemma 2.
Let $\varepsilon \in (0,1), \lambda \in \langle 0, 1-\varepsilon \rangle$. If the matrix $\mathbf{C}$ with elements $c_{i,j}$, $i, j = 1, \ldots, n$ is positive definite, then there exists (independently on $\lambda \in \langle 0, 1-\varepsilon \rangle$) a constant $\rho > 0$ such that $E_F g_\lambda(x, \xi)$ is a strongly concave, with the parameter ρ, function on X.

Proof.
First it follows from [14] that under the assumptions of Lemma 2 there exists a constant $\rho^* > 0$ such that $E_F g_2^*(x, \xi)$ is a strongly concave, with a parameter ρ^* function on X. Consequently, according to the Definition 1 we can see that g_λ, $\lambda \in \langle 0, 1-\varepsilon \rangle$ is a strongly concave, with the parameter $\rho = \varepsilon\rho^*$ (not depending on λ) function on X.

It is known that the Kolmogorov metric $d_K(F, G) := d_K(P_F, P_G)$ in the probability measures space is defined by

$$d_K(P_F, P_G) := d_K(F, G) = \sup_{z \in R^n} |F(z) - G(z)| \qquad (7)$$

To define the Wasserstein metric $d_{W_1}(F, G) := d_{W_1}(P_F, P_G)$ let $P(R^n)$ denote the set of all (Borel) probability measures on R^n,

$$M_1(R^n) = \left\{ \nu \in P(R^n) : \int_{R^n} \|z\| \nu(dz) < \infty \right\}.$$

If $D(\nu, \mu)$ denotes the set of those measures in $P(R^n \times R^n)$ whose marginal measures are ν and μ, then

$$d_{W_1}(\nu, \mu) = \inf \left\{ \int_{R^n \times R^n} \|z - \bar{z}\| \kappa(dz \times d\bar{z}) : \kappa \in D(\nu, \mu) \right\}, \quad \nu, \mu \in M_1(R^n).$$

(P_F denotes the probability measure coresponding to the fistribution function F.)

2. MULTIOBJECTIVE DETERMINISTIC OPTIMIZATION PROBLEMS

To investigate stability and empirical estimates in the multiobjective case (2) we recall a relationship between one–ojective and multiobjective deterministic problems, generally.

Let $f_i, i=1,\ldots,r$ be real–valued functions defined on R^n, $K \subset R^n$ be a nonempty set. We can introduce a deterministic multiobjective optimization problem in the form. Find

$$\max f_i(x),\ i=1,\ldots,r \text{ subject to } x \in K \tag{8}$$

Definition2.

The vector x^* is an efficient solution of the problem (8) if and only if there exists no $x \in K$ such that $f_i(x) \geq f_i(x^*)$ for $i=1,\ldots,r$ and such that for at least one i_0 one has $f_{i_0}(x) > f_{i_0}(x^*)$.

Definition3.

The vector x^* is a properly efficient solution of the multiobjective optimization problem (8) if and only if it is efficient and if there exists a scalar $\overline{M} > 0$ such that for each i and each $x \in K$ satisfying $f_i(x) > f_i(x^*)$ there exists at least one j such that $f_j(x^*) > f_j(x)$ and

$$\frac{f_i(x) - f_i(x^*)}{f_j(x^*) - f_j(x)} \leq \overline{M}.$$

Proposition2. [3]

Let $K \subset R^n$ be a convex set and let $f_i, i=1,\ldots,r$ be concave functions on K. Then x^0 is a properly efficient solution of the problem (8) if and only if x^0 is optimal in

$$\max_{x \in K} \sum_{i=1}^{r} \overline{\lambda}_i f_i(x) \text{ for some } \overline{\lambda}_1,\ldots,\overline{\lambda}_r > 0,\ \sum_{i=1}^{r} \overline{\lambda}_i = 1 \tag{9}$$

Remark1.

Let $r \geq 2$. Let, moreover, $x_{\overline{\lambda}}, \overline{\lambda} = (\overline{\lambda}_1,\ldots,\overline{\lambda}_r)$ be a solution of the problem (9). It follows from the proof of Theorem 1 of [3] that M corresponding to $x := x_{\overline{\lambda}}$ fulfils the relation $M = (r-1)\max_{i,j} \frac{\overline{\lambda}_i}{\overline{\lambda}_j}$.

Definition4.

If $K', K'' \subset R^n$ are two nonempty sets, then the Hausdorff distance of these sets $\Delta[K', K''] := \Delta_n[K', K'']$ is defined by

$$\Delta_n[K', K''] = \max[\delta_n(K', K''),\ \delta_n(K'', K')],$$

$$\delta_n(K', K'') = \sup_{x' \in K'} \inf_{x'' \in K''} \|x' - x''\|.$$

3. STABILITY AND EMPIRICAL ESTIMATES

To introduce new assertions we denote

$$\begin{aligned} x_F^\lambda &= \operatorname{argmax} E_F g_\lambda(x, \xi) \text{ everywhere when } x_F^\lambda \text{ is well-defined,} \\ \bar{X}_F &= \{x \in X : x \text{ is a properly efficient point of the problem (2)}\}, \\ \bar{X}_F^\varepsilon &= \{x \in X : x \text{ is a properly efficient point of the problem (2)} \\ &\quad \text{corresponding } \lambda \in \langle 0, 1-\varepsilon\rangle,\ \varepsilon \in (0,1)\} \end{aligned} \tag{10}$$

(For more detail explanation about the definition of $\bar{X}_F^\varepsilon$ see Remark 1.)

Theorem 1.

Let $\delta > 0, \varepsilon \in (0,1)$. If Z_F be a bounded set, $\lambda \in \langle 0, 1-\varepsilon\rangle$, then there exist two functions F_δ, F^δ defined on R^n and a constant L_1 such that for every n–dimensional distribution function $G(z)$

$$G(z) \in \langle F_\delta(z),\ F^\delta(z)\rangle,\quad z \in R^n$$

the following assertion is valid

$$\left|\sup_X E_F g_\lambda(x, \xi) - \sup_X E_G q_\lambda(x, \xi)\right| \le L_1 \delta \text{ for every } \lambda \in \langle 0, 1\rangle \tag{11}$$

If, moreover, the matrix **C** is positive definite, then there exists a constant $L_1' := L_1'(\varepsilon)$ such that also

$$\|x_F^\lambda - x_G^\lambda\|^2 \le L_1'\delta \text{ for every } \lambda \in \langle 0, 1-\varepsilon\rangle, [\Delta(X_F^\varepsilon,\ G_G^\varepsilon)]^2 \le L_1'\delta \tag{12}$$

(Z_F denotes the support of the probability measure P_F.)

Sketch of the Proof.

Employing the approach of Theorem 2 of [8] and Lemma 1 we can see that the assertion (11) is valid.. Furthermore, first assertion of (12) follows from the assertion (11), Lemma 2 and Proposition 1. To obtain the second assertion of (12) we employ the assertion of Proposition 2 and Remark 1.

Remark 2.

Evidently, if the probability measure P_F is absolutely continuous w.r.t. the Lebesgue measure in R^n, then employing the assertion of Theorem 1, the stability results w.r.t. Kolmogorov metric can be obtained; for more details of this approach see e.g. [8]. The support Z_F must be bounded for it. In [13] an approach is introduced under that this restriction can be omitted.

Theorem 2.

Let $\varepsilon \in (0,1)$. If Z_F is a bounded set, $\lambda \in \langle 0, 1-\varepsilon \rangle$, distribution function G fulfils the assumption A.1, then there exists a constant L_2 such that the following assertion is valid.

$$\left| \sup_X E_F g_\lambda(x,\xi) - \sup_X E_G g_\lambda(x,\xi) \right| \le L_2 d_{W_1}(P_F, P_G) \text{ for every } \lambda \in \langle 0, 1 \rangle. \tag{13}$$

If, moreover, the matrix $\mathbf{C}$ is positive definite, then there exists a constant L_2' such that also

$$\begin{aligned} \left\| x_F^\lambda - x_G^\lambda \right\|^2 &\le L_2' d_{W_1}(P_F, P_G) \quad \text{for every } \lambda \in \langle 0, 1-\varepsilon \rangle \\ \left[\Delta\left(X_F^\varepsilon, G_G^\varepsilon \right) \right]^2 &\le L_2' d_{W_1}(P_F, P_G) \end{aligned} \tag{14}$$

Sketch of the Proof.

The idea of the Proof of Theorem 2 is very similar to the main idea of the proof of Theorem 1. However instead of the results of the work [8] we employ in this case the results of the paper [11].

Theorem 1 and Theorem 2 deal with the case when the theoretical probability measure is replaced by a deterministic one. However very often the theoretical probability measure must be replaced by an empirical one. Employing the large deviations technique we obtain the next assertion.

Theorem 3.

Let $t > 0, \varepsilon \in (0,1)$, $\lambda \in \langle 0, 1-\varepsilon \rangle$. Let, moreover, Z_F be a bounded set. If F_N, $N = 1, \ldots$ is an empirical distribution function determined by an independent random sample $\{\xi^i\}_{i=1}^N$ corresponding to the distribution function F, then there exist constants k_1, $K_1 > 0$ such that

$$\begin{aligned} P\Big\{ &\left| \sup_X E_{F^N} g_\lambda(X,\xi) - \sup_X E_F g_\lambda(x,\xi) \right| > t \\ &\text{for at least one } \lambda \in \langle 0, 1 \rangle \Big\} \le k_1 \exp\{-K_1 N t^2\}, \quad N = 1, \ldots \end{aligned} \tag{15}$$

If moreover the matrix $\mathbf{C}$ is positive definite, then there exist constants $k_2, K_2 \ge 0$ such that also

$$P\left\{ \left\| x^{\lambda}_{F^N} - x^{\lambda}_{F} \right\|^2 > t \text{ for at least one } \lambda \in \langle 0, 1-\varepsilon \rangle \right\} \le k_2\, exp\left\{-K_2 N t^2\right\},$$

$$P\left\{ \left[\Delta\left[X^{\varepsilon}_{F_N},\ X^{\varepsilon}_{F} \right]\right]^2 > t \right\} \le k_2 \exp\left\{-K_2 N t^2\right\}, \quad N = 1, \ldots \tag{16}$$

Sketch of the Proof.

To prove Theorem 3 we employ, first, the assertion of Lemma 1 to see that $g_{\lambda}(x, z)$ is a Lipschitz function on X with a Lipschitz constant not depending on z and $\lambda \in \langle 0, 1-\varepsilon \rangle$. Moreover, according to the special form of the function $g_{\lambda}(x, z)$ it is easy to see that we can apply the assertion of Theorem 2 of [5] to get the relation (15). The relation (16) can be obtained very similarly.

4. CONCLUSION

We have introduced results on the stability and empirical estimates concerning portfolio optimal selection problem. To this end we omit usual assumption about the normal distribution. Consequently to this, we have assumed a bounded probability measure support. Surely it would be very useful to deal with the stability (considered w.r.t. parameter λ) of the problem (4) as well as to deal with the case of general support. However the both these investigations are over the possibilities of this contribution.

Acknowledgement. This research was supported by the Grant Agency of the Czech Republic under Grants 402/04/1294, 402/02/1015 and the Grant Agency of AS CR under Grant A 7075202.

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DYNAMICAL MACROECONOMIC MODELS FROM THE KEYNESIAN, WALRASIAN AND CLASSICAL POINT OF VIEW

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Abstract: In this article we compare properties of Keynesian, Walrasian and Classical macroeconomic models. We start with an extended dynamical IS-LM neoclassical model generating behaviour of the real product, real interest rate, expected inflation and the price level over time. Limiting behaviour, stability, existence of limit cycles and other specific features of this model will be compared with those of Walrasian and Classical models. [1]

Keywords: Macroeconomic models, Keynesian, Walrasian and Classical model, nonlinear differential equations, linearization, asymptotical stability.

1. INTRODUCTION AND NOTATIONS

The need of dynamic macroeconomics was already recognized some fifty years ago, when the first models of business cycles were studied (e.g. Frisch [3], Hicks [5], [6], Kalecki [8], [9], Samuelson [11]). In those contributions it was shown that the dynamic interaction of the multipliers and the accelerator in Keynesian macroeconomic models can generate fluctuations in output and employment. Those theories focused mainly on the dynamics of product markets.

In this article, we describe commodity market, money market and the equilibrium of production sector from Keynesian, Walrasian and Classical point of view. We will prefer the continuous-time description of the considered macroeconomic systems, i.e. monetary and price dynamics is then described by systems of differential equations.

To begin with, we present the basic description of dynamic macroeconomic system employing the IS-LM approach. This approach enables to enclose also the money market and moreover, introducing a Phillips curve enables to extend the dynamic models also to labour market (e.g. Fischer [1], Sargent [12], [13], Tobin [19]).

1 This research was supported by the Grant Agency of the Academy of Sciences of the Czech Republic under Grant A7075202 and by the Grant Agency of the Czech Republic under Grant 402/02/1015.

Let (in continuous time $t \geq 0$) $Y(t)$, $S(\cdot,\cdot)$ and $I(\cdot,\cdot)$ denote the real product, savings and real investments of the considered economy respectively. Recall that for the nominal interest $R(t)$ it holds $R(t) = r(t) + \pi^e(t)$ where $r(t)$ is the real rate of interest and $\pi^e(t)$ the expected inflation, in contrast to the real inflation $\pi(t)$. The dynamic IS model (i.e. Investment-Saving Model) is then given by the following differential equation (see e.g. [18]) $\dot{Y} = \alpha\{I[Y(t), r(t)] - S[Y(t), r(t)]\}$ or on taking logarithms by

$$\frac{dy(t)}{dt} = \alpha\{i[y(t), r(t)] - s[y(t), r(t)]\} \tag{1.1}$$

where $y(t) = \ln Y(t)$ and $i(\cdot,\cdot) = \dfrac{I(\cdot,\cdot)}{Y(\cdot,\cdot)}$, $s(\cdot,\cdot) = \dfrac{S(\cdot,\cdot)}{Y(\cdot,\cdot)}$, is the so-called propensity to invest, to save respectively. Observe that for an equilibrium point $Y(t) \equiv Y^*$, $y(t) \equiv y^*$, $r(t) \equiv r^*$ we have $I(Y^*, r^*) = S(Y^*, r^*)$ or $i(y^*, r^*) = s(y^*, r^*)$.

Denoting by $p(t)$ the price level at time t, the dynamics of the money market (LM model) is described by the following differential equation

$$\begin{aligned} \frac{d\,r(t)}{dt} &= \beta\left\{\ell[y(t), R(t)] - \ln\frac{M^s}{p(t)}\right\} = \\ &= \beta\{\ell[y(t), r(t) + \pi^e(t)] - (m^s - \bar{p}(t))\} \end{aligned} \tag{1.2}$$

where $\ell[y(t), R(t)] = \ln[L(Y(t), R(t))]$, $m^s = \ln M^s$, $\bar{p}(t) = \ln p(t)$; $L(\cdot,\cdot)$ and M^s is reserved for demand for money and money supply respectively. In (1.1), (1.2) alpha, beta are positive constants signifying the speed of adjustment of the respective market.

To obtain a complete dynamic model of the economy we need to include equations for expected inflation $\pi^e(t)$ and the price level $p(t)$.

According to Tobin [20], for $\pi^e(t)$ the following adaptive equation is valid

$$\frac{d\,\pi^e(t)}{dt} = \gamma[\pi(t) - \pi^e(t)] \tag{1.3}$$

where γ is the coefficient of adaptation and $\pi^e(t)$ is the real inflation.

Recalling that $\pi(t) = \dfrac{\dot{p}(t)}{p(t)} = \dfrac{d\,\bar{p}(t)}{dt}$, from (1.3) we immediately get

$$\frac{d\,\pi^e(t)}{dt} = \gamma\left[\frac{d\,\bar{p}(t)}{dt} - \pi^e(t)\right]. \tag{1.4}$$

For what follows we need to express $\frac{d\,\bar{p}(t)}{dt}$. To this end shall assume that the development of the price level $p(t)$ over time is in accordance with changes of the so-called cost function $C(y(t))$. In particular, the well-known condition of profit maximization $p(t) - \frac{dC(y)}{dy}$ is the base for the following adjustment formula for $p(t)$ (delta is a constant):

$$\frac{d\,\bar{p}(t)}{dt} = \delta\left[1 - e^{-\bar{p}(t)}\,\frac{dC(y)}{dy}\right] \tag{1.5}$$

In fact, the above formula is in accordance with the traditional theory of perfectly competitive firms (see e.g. [10] and as such is interpreted in many treatises on monetary and price dynamics (cf. e.g. [2]).

2. DYNAMIC MACROECONOMIC MODELS: GENERAL VIEW

In what follows we shall use shorthand notations only, i.e. we replace $\frac{d\,\bar{p}(t)}{dt}$ by $\dot{\bar{p}}$ similarly for the time derivatives $\dot{y}$, $\dot{r}$, $\dot{\pi}^e$, and $\frac{dC(y)}{dy}$ is replaced by $C'(y)$. More over, we shall often omit the argument *t*. Hence, (cf. (1.1), (1.2), (1.4), (1.5)) using such a model the system describing an economy from the Keynesian point of view has the following form:

$$\begin{aligned}
\dot{y} &= \alpha\,[i(y,r) - s(y,r)] \\
\dot{r} &= \beta\left[\ell\left(y, r+\pi^e\right) - \left(m^s - \bar{p}\right)\right] \\
\dot{\pi}^e &= \gamma\left(\bar{p} - \pi^e\right) \\
\dot{\bar{p}} &= \delta\left[1 - e^{-\bar{p}}C'(y)\right]
\end{aligned} \tag{2.1}$$

where $i(y,r)$, $s(y,r)$, $\ell(y, r+\pi^e)$, and $C(y)$ are real ivestment, real savings, real money demand and cost functions respectively, depending on production *y*, real rate of interest *r*, (expected) inflation π^e and the price level *p*.

The Walrasian model has different price and commodity dynamics from the Keynesian model. Comparing the Walrasian and Keynesian model observe that in the Keynesian model the price dynamics is controlled by costs as it is usual in Keynesian economics and commodity dynamics depends on the disequilibrium on commodity markets.

The Walrasian model consists of four differential equations that formalize the (commodity) price level, interest rate, production and expected inflation dynamics in the following manner:

$$
\begin{aligned}
\dot{\bar{p}} &= \alpha\left[i(y, r) - s(y, r)\right] \\
\dot{r} &= \beta\left[\ell\left(y, r + \pi^{e}\right) - \left(m^{s} - \bar{p}\right)\right] \\
\pi^{e} &= \gamma\left(\dot{\bar{p}} - \pi^{e}\right) \\
\dot{y} &= \delta\left[1 - e^{-\bar{p}}C'(y)\right]
\end{aligned}
\tag{2.2}
$$

Classical models that describe (commodity) price level, interest rate, production and expected inflation dynamics have similar structure of right hand sides of differential equations, but left hand sides are permuted as it is shown below:

$$
\begin{aligned}
\dot{r} &= \alpha\left[i(y, r) - s(y, r)\right] \\
-\dot{\bar{p}} &= \beta\left[\ell\left(y, r + \pi^{e}\right) - \left(m^{s} - \bar{p}\right)\right] \\
\dot{\pi}^{e} &= \gamma\left(\dot{\bar{p}} - \pi^{e}\right) \\
\bar{y} &= \delta\left[1 - e^{-\bar{p}}C'(y)\right]
\end{aligned}
\tag{2.3}
$$

Just introduced models are the base for establishment of macroeconomic models of price and monetary dynamics.

Comparing above systems of differential equations for Keynesian, Walrasian and Classical models, we can conclude that for the study of macroeconomic systems, the general object of our interest is a system described (in the continuous-time setting) by the system of nonlinear first-order (autonomous) differential equations written in a condensed form as

$$
\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}\left(\mathbf{x}(t)\right) \text{ with } \mathbf{x}(t_0) = \mathbf{x}^0 \tag{2.4}
$$

where the state of the system $\mathbf{x}(t) \in \mathbf{R}^4$ and $\mathbf{f}(\cdot)$ is a continuously differentiable vector function from $\mathbf{R}^4$ to je $\mathbf{R}^4$ defined on an open subset je $\mathbf{R}^4$. Observe that ($[.]^T$ denotes the transpose)

$$
\mathbf{f}\left(\mathbf{x}(t)\right) = \left[f_1\left(\mathbf{x}(t)\right), f_2\left(\mathbf{x}(t)\right), f_3\left(\mathbf{x}(t)\right), f_4\left(\mathbf{x}(t)\right)\right]^T
$$

$$
\begin{aligned}
\mathbf{x}(t) &= \left[y(t), r(t), \pi^{e}(t), p(t)\right]^T && \text{for the Keynesian model} \\
&= \left[p(t), r(t), \pi^{e}(t), y(t)\right]^T && \text{for the Walrasian model} \\
&= \left[r(t), p(t), \pi^{e}(t), y(t)\right]^T && \text{for the Classical model}
\end{aligned}
$$

Recall that $\mathbf{x}^* \in \mathbf{R}^N$ is the equilibrium point of the above system, iff $\mathbf{f}(\mathbf{x}^*) = 0$ and $\mathbf{x}^*$ is said to be (asymptotically) locally stable if every solution $\mathbf{x}(t)$ of the considered system, starting sufficiently close to $\mathbf{x}^*$ converges to $\mathbf{x}^*$ as $t \to \infty$. Similarly, $\mathbf{x}^*$ is said to be (asymptoticaly) globally stable if $\mathbf{x}(t) \to \mathbf{x}^*$ regardless the starting point $\mathbf{x}^0$. It is well known (cf. e.g. [4]or [18]) that an equilibrium point (and also a stable point) of the system need not exist, hence the system is unstable. Recall that having found equilibrium points, the system need not convergence to some or any of the equilibrium points (in the latter case the system is unstable). Furthermore, if $\mathbf{f}(\mathbf{x})$ is nonlinear, the considered unstable system can also exhibit limit cycles (i.e. its trajectory remains in a bounded region). In words, in contrast to unstability or limit cycles, stability is equivalent to monotone or oscillating convergence toward the equilibrium point.

Inspecting (2.1), (2.2), (2.3) it is clear that in the equilibrium $\pi^{e*} = 0$ and the equilibrium point

$$\begin{aligned} \mathbf{x}^* &= \left[y^*, r^*, 0, \bar{p}^*\right]^T \quad \text{for the Keynesian model} \\ &= \left[\bar{p}^*, r^*, 0, y^*\right]^T \quad \text{for the Walrasian model} \\ &= \left[r^*, \bar{p}^*, 0, y^*\right]^T \quad \text{for the Classical model} \end{aligned}$$

must be the same for all considered models and can be ontained by solving the following set of equations:

$$\begin{aligned} i(y, r) &= s(y, r) \\ \ell\left(y, r + \pi^e\right) &= m^s - \bar{p} \\ e^{-\bar{p}} &= C'(y) \end{aligned} \tag{2.5}$$

3. APPROXIMATION AND LINEARIZATION OF THE MODELS

To find an analytical form of the output $y(t) = \ln Y(t)$, real interest rate $r(t)$, expected inflation $\pi^e(t)$ and the price level $p(t)$ we need to assume that the functions $i(\cdot,\cdot)$, $s(\cdot,\cdot)$, $C(\cdot)$ are of a specific analytical form.

As usual, the functions $s(\cdot,\cdot)$ as well as demand for money $\ell(y, R)$ can be well approximated by linear functions, whereas it is necessary to approximate $i(\cdot,\cdot)$ and sometimes also $C(\cdot)$ by suitable nonlinear functions.

In what follows, we assume that savings $S(Y(t), r(t))$ can be well approximated by the following expression

$$S(Y(t), r(t)) = Y(t) \cdot [s_0 + s_1 \cdot y(t) + s_2 \cdot r(t)] \text{ with } s_0 < 0, \text{ and } s_1, s_2 > 0. \tag{3.1}$$

Hence the propensity to save $s(y(t), r(t)) = S(Y(t), r(t))/Y(t)$ can be written as

$$s(Y(t), r(t)) = s_0 + s_1 \cdot y(t) + s_2 \cdot r(t) \tag{3.2}$$

Similarly, the demand for money is described by the traditional Keynesian demand-for-money function being in the following form

$$\begin{aligned} \ell(y(t), R(t)) &= \ell_0 + \ell_1 y(t) - \ell_2 R(t) - \ell_3 \pi^e(t) = \\ &= \ell_0 + \ell_1 y(t) - \ell_2 \left[r(t) + \pi^e(t) \right] - \ell_3 \pi^e(t) \end{aligned} \tag{3.3}$$

where the parameters $\ell_i > 0,\ i = 0,1,2,3$ are given.

On the orther hand, it is convenient to assume that the propensity to invest $i(y(t), r(t))$ is a product of $\frac{1}{r(t)+1}$ and the so-called logistic function. Hence the propensity to invest is assumed to be given analytically as

$$i(y(t), r(t)) = \frac{1}{r(t)+1} \cdot \frac{k}{1 + be^{-ay(t)}} \tag{3.4}$$

where the parameters $k, a > 0$ and b is an arbitrary real number.

Similarly, we shall assume that the cost function $C(\cdot)$ is also a logistic function given analytically as

$$C(y(t)) = \frac{h}{1 + de^{-cy(t)}} \tag{3.5}$$

where the parameters $h, c > 0$ and d is an arbitrary real number. Hence

$$\frac{dC(y)}{dy} = \frac{h}{\left(1 + de^{-cy(t)}\right)^2} (-cdy) \tag{3.6}$$

and we can assume that the „central" part of $C(y(t))$ can be well approximated by a linear function

$$C(y(t)) = d_0 + d_1 y(t) \tag{3.7}$$

Since $\pi^{e*} = 0$ to calculate the values y^*, r^*, p^*, on inserting (3.2), (3.3), (3.4) and (3.7) into (2.5) we have

$$\frac{1}{r^* + 1} \cdot \frac{k}{1 + be^{-ay^*}} = s_0 + s_1 y^* + s_2 r^* \tag{3.8}$$

$$\ell_0 + \ell_1 y^* - \ell_2 r^* = m^s - \bar{p}^* \tag{3.9}$$

$$\bar{p}^* = -\ln d_1 \overset{def}{=} -\bar{d}_1 \tag{3.10}$$

In virtue of (3.10) from (3.8), (3.9) the equilibrium values y^*, r^* can be found as a solution to

$$\frac{k}{1+be^{-ay^*}} = \left(s_0 + s_1 y^* + s_2 r^*\right)\left(1+r^*\right) \tag{3.11}$$

$$r^* = \frac{1}{\ell_2}\left(\ell_0 - \left(m^s + \bar{d}_1\right) + \ell_1 y^*\right) \Leftrightarrow y^* = \frac{1}{\ell_1}\left(\left(m^s + \bar{d}_1\right) - \ell_0 + \ell_2 r^*\right) \tag{3.12}$$

From (3.11), (3.12) we get

$$\left[s_0 + s_2\left(\frac{\ell_0}{\ell_2} - \frac{m^s + \bar{d}_1}{\ell_2}\right) + \left(s_1 + s_2\frac{\ell_1}{\ell_2}\right)y^*\right]\cdot$$
$$\cdot\left[1 + \frac{\ell_0}{\ell_2} - \frac{m^s + \bar{d}_1}{\ell_2} + \frac{\ell_1}{\ell_2}y^*\right] = k\cdot\frac{1}{1+be^{-ay^*}} \tag{3.13}$$

Hence finding the solution to (3.13) and inserting this value into (3.12) we immediately get the pair of equilibrium points y^*, r^*. We can observe that:

1.) (The RHS of (3.13) is the so-called logistic function (an increasing function having an inflection point at $y = \frac{1}{a}\ln b$ that is convex in the interval $\left(0, \frac{1}{a}\ln b\right)$ and concave in $\left(\frac{1}{a}\ln b, \infty\right)$;
2.) (The LHS of (3.13) is a quadratic function (in fact, for real-life models this function differs only slightly from a straight line).

Hence there exist at most three, in real models usually only one, pair(s) of equilibrium points y^*, r^* for $y \geq 0$.

More insight in the properties of the equilibrium poinst, especially with respect to the stability, can be obtained by linearization around the neighbourhood of the equilibrium point $\left(y^*, r^*, \pi^{e*}, \bar{p}^*\right)$ with $\pi^{e*} = 0$.

Linearizing we conclude that (2.1), (2.2), (2.3) can be written following common form

$$\begin{bmatrix} \dfrac{d\left(x_1(t)-x_1^*\right)}{dt} \\ \dfrac{d\left(x_2(t)-x_2^*\right)}{dt} \\ \dfrac{d\left(x_3(t)-x_3^*\right)}{dt} \\ \dfrac{d\left(x_4(t)-x_4^*\right)}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_1(\mathbf{x})}{\partial x_2} & \dfrac{\partial f_1(\mathbf{x})}{\partial x_3} & \dfrac{\partial f_1(\mathbf{x})}{\partial x_4} \\ \dfrac{\partial f_2(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_2(\mathbf{x})}{\partial x_2} & \dfrac{\partial f_2(\mathbf{x})}{\partial x_3} & \dfrac{\partial f_2(\mathbf{x})}{\partial x_4} \\ \dfrac{\partial f_3(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_3(\mathbf{x})}{\partial x_2} & \dfrac{\partial f_3(\mathbf{x})}{\partial x_3} & \dfrac{\partial f_3(\mathbf{x})}{\partial x_4} \\ \dfrac{\partial f_4(\mathbf{x})}{\partial x_1} & \dfrac{\partial f_4(\mathbf{x})}{\partial x_2} & \dfrac{\partial f_4(\mathbf{x})}{\partial x_3} & \dfrac{\partial f_4(\mathbf{x})}{\partial x_4} \end{bmatrix} \cdot \begin{bmatrix} x_1(t)-x_1^* \\ x_2(t)-x_2^* \\ x_3(t)-x_3^* \\ x_4(t)-x_4^* \end{bmatrix} \quad (3.14)$$

where

$$\left[x_1(t), x_2(t), x_3(t), x_4(t)\right] = \left[y(t), r(t), \pi^e(t), \bar{p}(t)\right] \text{ for the Keynesian model}$$

$$= \left[\bar{p}(t), r(t), \pi^e(t), y(t)\right] \text{ for the Walrasian model}$$

$$= \left[r(t), \bar{p}(t), \pi^e(t), y(t)\right] \text{ for the Classical model}$$

The equilibrium point $\left(x_1^*, x_3^*, x_3^*, x_4^*\right)$ is stable if and only if the system (3.14) is stable, i.e. if all eigenvalues of the matrix of the system have negative real parts.

In particular, on employing (3.8), (3.9), (3.10) for the Keynesian model we have:

$$\begin{bmatrix} \dfrac{d\left(y(t)-y^*\right)}{dt} \\ \dfrac{d\left(r(t)-r^*\right)}{dt} \\ \dfrac{d\left(\pi^e(t)\right)}{dt} \\ \dfrac{d\left(p(t)-p^*\right)}{dt} \end{bmatrix} = \begin{bmatrix} \alpha\left(D_y - s_1\right) & \alpha\left(D_r - s_2\right) & 0 & 0 \\ \beta\ell_1 & -\beta\ell_2 & -\beta\left(\ell_2+\ell_3\right) & \beta \\ 0 & 0 & -\gamma & \gamma d_1^2 \\ 0 & 0 & 0 & \delta d_1^2 \end{bmatrix} \cdot \begin{bmatrix} y(t)-y^* \\ t(t)-r^* \\ \pi^e(t) \\ p(t)-p^* \end{bmatrix} \quad (3.15)$$

where

$$D_y = \left.\frac{1}{1+r}\right|_{r=r^*} \cdot \left.\frac{\partial}{\partial y}\frac{k}{1+be^{-ay(t)}}\right|_{y=y^*}, \quad D_r = \left.\frac{\partial}{\partial r}\frac{1}{1+r}\right|_{r=r^*} \cdot \left.\frac{k}{1+be^{-ay(t)}}\right|_{y=y^*}.$$

So we have

$$D_y = \frac{1}{1+r^*} \cdot \frac{kabe^{-ay^*}}{\left(1+be^{-ay^*}\right)^2}, \qquad D_r = -\frac{1}{\left(1+r^*\right)^2} \cdot \frac{k}{1+be^{-ay^*}}$$

where from (3.13)

$$k = \left[s_0 + s_2 \left(\frac{\ell_0}{\ell_2} - \frac{m^s + \bar{d}_1}{\ell_2} \right) + \left(s_1 + s_2 \frac{\ell_1}{\ell_2} \right) y^* \right] \cdot \left[1 + \frac{\ell_0}{\ell_2} - \frac{m^s + \bar{d}_1}{d_1 \ell_2} + \frac{\ell_1}{\ell_2} y^* \right] \cdot$$
$$\cdot \left[1 + b e^{-a y^*} \right]$$

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STOCHASTIC COSTS IN THE TRAVELING SALESMAN PROBLEM

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Abstract Solving the traveling salesman problem we often need to use the time as the objective function. But the time spent by a single realization of transport on a given route may differ depending on the actual traffic situation. So it has a stochastic character. Using this point of view, the evaluation of a particular route consists of two components: the proper time (meaning e.g. its expected value) of the transportation and the reliability (probability) that a given time will not be exceeded. So we have two criteria, i.e. a multiplecriteria task is obtained.

However, the methods for solving the traveling salesman problem cannot usually be associated with the stochastic approaches and the multiplecriteria methods. This contribution shows, that the iteration methods for the traveling salesman problem are suitable to be modified for this purpose and one version is applied here on several test instances.

Keywords: travelling salesman problem (TSP), stochastic models, multiplecriteria programming, iteration methods for the TSP.

1. INTRODUCTION

The original travelling salesman problem (TSP) is the task on how to make a circuit of given points (cities, places etc.) with the minimum cost (or distance, assuming that the cost depends proportionally on distance). In practice, the requirement to realize transportation by a given deadline or in the shortest possible term often appears, e.g. in activities of the delivery firms or in the case of agricultural or grocery products which quickly go bad etc. The transportation time is subject to random influences, e.g. traffic jams and accidents, weather conditions including elemental calamities, waiting for the check out at the state border etc. Thus the necessary time for passing a given route may be seen as a stochastic quantity and it is well-founded to interest in the TSP with the criteria of the stochastic character. This contribution contains a proposal of an approach for solving such problems and a trial to test it.

2. HANDLING THE STOCHASTICITY OF COSTS

The distribution of probabilities for the time of transportation between a particular pair of points (cities) is hard to determine. Thus we apply an analogous approach like in the PERT method. Let us suppose that for each of these pairs the transportation time is a random value with beta-distribution and the most probable (i.e. modus), optimistic and pessimistic time are known. Let $\mathbf{C}^m, \mathbf{C}^a, \mathbf{C}^b$ denote the matrices of these most probable, optimistic and pessimistic estimations, respectively. Let $\mathbf{C}$ denotes the matrix of expected values of these times and $\mathbf{S}$ the matrix of their dispersions. So if the random value representing the time spent by transportation from the point i to the point j is t_{ij}, then the elements of matrices $\mathbf{C}$ and $\mathbf{S}$ means $c_{ij} = \mathrm{E}\left[t_{ij}\right]$ and $s_{ij} = \sigma^2\left[t_{ij}\right]$ and using the standard notation for expected value and dispersion.

We can compute matrices $\mathbf{C}$ and $\mathbf{S}$ from $\mathbf{C}^m$, $\mathbf{C}^a$ and $\mathbf{C}^b$ applying well known relations among the characterizations of beta-distribution:

$$c_{ij} = \frac{c_{ij}^a + 4c_{ij}^m + c_{ij}^b}{6},$$

$$s_{ij} = \frac{\left(c_{ij}^b - c_{ij}^a\right)^2}{36}.$$

Now let us consider a particular feasible solution to our problem, i.e. some circular route going through all given required points. We can look onto at tour like a project solvable by the PERT method. Activities are single legs between two points in succession. The network representing this situation looks very simple: it is a path. Assuming that cost is an independent quantity, the expected value or the dispersion of the objective function for a given solution is equal to the sum of the expected values or of the dispersions, respectively, i.e.

$$\mathrm{E}\left[t_0\right] = \sum_{(i,j)\in S} c_{ij},$$

$$\sigma\left[t_0\right] = \sum_{(i,j)\in S} s_{ij},$$

where S is the set of all such (ordered) pairs of points (cities) that the latter point immediately follows the former one on the subjected route.

If we want to catch to go through the whole route until a given time d, we can use the relation for the standardized variation u of random value t_0 with the normal distribution

$$u = \frac{d - \mathrm{E}\left[t_0\right]}{\sqrt{\sigma^2\left[t_0\right]}}$$

and find the probability of keeping this term in the table of the distribution function of the standardized normal distribution.

3. MULTIPLECRITERIA NATURE OF THE STOCHASTIC TSP

Any feasible solution of the stochastic TSP may be evaluated according to two criteria: the proper time (e.g. its expected value, the most probable value etc.) and the reliability (the probability that a given required time for passing the route will not be exceeded). The situation is even more complicated than in the case of the classical multiplecriteria programming. The values of the criteria depend reciprocally to one of another. One of them may be chosen and the remaining one computed for any such choice. Of course, this property does not imply any obstruction for the application of the replacement of one of these criteria by the constraint.

This method of replacement of the objective function (criterion) by the constraint transforms the task with two criteria to one with only one criterion by the following way: Let F_1 be the criterion which is to be replaced by a constraint. Let it be a maximized criterion, f_{max} its optimal value and f_{min} its minimal value. Then some value $f^* \in (f_{min}, f_{max})$ is chosen (as the compromise value which the user is ready to accept) and the criterion F_1 is replaced by the constraint $F_1 \geq f^*$ added to the coinstrant system of the model. The latter criterion, say F_2, remains to be optimized. This method is described more preciously in [1].

Applying this method for the stochastic TSP, we replace the criterion of reliability by the constraint. The required reliability f^* having been chosen, the time of passing any feasible solution is uniquely determined. In the original version of the method the new added constraint is an inequality. Nevertheless, because for any particular tour, decreasing the required time for going through it, the reliability decreases, too, an equation may be used instead the inequality. Thus the use of this method is correct.

4. ITERATION METHODS FOR SOLVING THE TSP

For solving the stochastic TSP iteration methods for the TSP, e.g 2-opt, 3-opt, Or method (Or-opt), Lin-Kerninghan method, may be modified. In the original version each of these methods defines a neighbourhood, it means a set of the neighbour feasible solutions, for every feasible solution. This set is either a given constant (in the most of these methods) or the method creates it in each iteration according to the properties of the solutions which has already been included into the neighbourhood (in Lin-Kerninghan method). These methods compare the initial solution for a given iteration with all solutions in its neighbourhood and if there are some better ones (with the lower value of the objective function) among them, then one of them is chosen as the initial for the next iteration, if not, then the computation is finished and this solution is considered the final one. A large overview of methods for solving the TSP in details (including these iteration methods) is given in [2].

Having given the stochastic TSP and using the approach described in chapter 2, we can appreciate the quality of solutions in the neigbourhood of one currently being initial in the used iteration method. We can do it even according both the criteria mentioned in chapter 3 in the same time. This is the way, to modify these iteration methods for the stochastic TSP.

We use a modification of the 2-opt for testing this approach in the next chapter. Now we can describe its version for the classical TSP with the symmetric cost matrix. Let the initial cyclic tour is given and $\tau(i)$ denotes the point immediately following after the point *i*. Find a pair of different points *i*, *j* so that the expression $c_{i\tau(i)} + c_{j\tau(j)} - c_{ij} - c_{\tau(i)\tau(j)}$ acquires the maximum possible value. If it is nonnegative then it is impossible to make the route better by changing two legs on it. In this case the algorithm is terminated and the tour is declared to be a resulting solution. In the opposite case the route is repaired as follows: Using notation

$$\tau(i) = l_1,\ j = l_u,\ \tau(l_t) = l_{t+1} \ \text{ for } \ t = 1, \ldots, u-1$$

assign

$$\tau(i) := j,\ \tau(l_1) = \tau(j),\ \tau(l_t) = l_{t-1} \ \text{ for } \ t = 2, \ldots, u$$

Then apply the procedure above with this new tour (and repeat it until the value of the above introduced expression is nonnegative).

5. TEST COMPUTATIONS AND THEIR RESULTS

Test instances of the stochastic TSP were randomly generated the following way: In a circle with the radius 100 km lying in a Euclidian plane E_2 twelve points were randomly chosen with the uniform probability distribution. Then the accessibility (e.g. altitude, the quality and ”straightness” of roads in its surroundings etc.) was determined for each point, again uniformly randomly and from an interval with the lower bound equal to zero and the upper bound directly proportional to the distance of the point from the centre of the circle. This value was taken as the third coordinate of the point in a 3-dimensional Euclidian space E_3 and the (road) distance between any two points was defined as their Euclidian distance in this space E_3. For each point the usable average speed of transport in its surroundings was generated uniformly randomly from the interval from 40 to 110 km per hour. The average speed on the road between two points was computed as the harmonic average of the speeds in their surroundings. The time spent by using this speed was considered to be the most probable and so the matrix $\mathbf{C}^{\mathrm{m}}$ (cf. chapter 2) was obtained. The matrix of the optimistic estimates of time was set $\mathbf{C}^{\mathrm{a}} = 0.9 \cdot \mathbf{C}^{\mathrm{m}}$. Elements of $\mathbf{C}^{\mathrm{b}}$ were generated uniformly randomly from the intervals which lower bound for each route between two points was the corresponding element of $\mathbf{C}^m$ and the upper bound was the time necessary for passing the shortest detour through some third point.

Five instances were generated by this way. They were solved as classical TSP's by Christofides method using the cost matrix $\mathbf{C}$. This algorithm was chosen because it has the best known mathematically proven ratio z_{comp}/z_{opt} among all methods for solving TSP, where z_{comp} and z_{opt} denote the computed and the (mathematically) optimal value, respectively.

Then the required reliability was set to 95 p.c. This from this moment times

$$d = \mathrm{E}\left[t_0\right] + u\sqrt{\sigma^2\left[t_0\right]}$$

(using notation in chapter 2) were considered to be costs (i.e. $u \approx 1{,}65$) for the given reliability and the 2-opt algorithm was applied.

From these five cases in all of them a route was found, which was accessible with 95 p.c. probability in a shorter time than the initial one found by the Christofides method using the cost matrix $\mathbf{C}$.

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SINGLE-PERIOD INVENTORY ANALYSIS - DIFFERENCES BETWEEN MATHEMATICAL MODELS AND SIMULATION MODELS

MARTINA KUNCOVÁ

1. INTRODUCTION

Inventory optimisation and inventory control belong to the most important fields in business companies. In case no appropriate software is available, managers must estimate the order quantity and order time themselves, mostly on the basis of their individual knowledge of customers' behaviour. Nowadays, when it is necessary to be very quick and perfect in reaction to the customer's demand, this approach seems to be insufficient. Of course the best way is to use some special software, but corresponding cost of it may be quite significant and additionally, the installation of such systems may require radical changes in company's structure. It also can take a year or more to install. Even big firms should consider buying the expensive software specialized just on the partial problems in company's operations. However, as it will be shown later, using standard spreadsheets (e.g. MS Excel) is suitable access to solve the company's problems. There are two possibilities how to manage the inventory - analytically, using mathematical model or to use a simulation. The comparison of these two approaches is discussed in this paper on case of the single-period decision inventory model.

2. THE NEWSBOY PROBLEM - DEFINITION

The single-period decision inventory model is usually called "The Newsboy Problem" or "The Christmas Tree Problem" and it belongs to the stochastic inventory models. The model refers to the situations in which only one order is placed for the considered time period. At the end of the period the inventory might be out of stock or there might be a surplus of units and for these cases there is a penalty associated with overstock or stockout. This is the case of selling seasonal goods such as Christmas trees, Christmas cards, bread, fruits, flowers, newspapers, etc In case of the newsboy who sells newspapers on the street, the demand is uncertain, and he must decide how many papers to buy from his supplier. If he buys too many papers, the unsold ones have no value at the end of the day or week (or nearly no value) and it means a loss for him. If he buys less papers than there will be demanded, he has lost the opportunity of making higher profit.

3. MATHEMATICAL MODELS FOR THE "NEWSBOY PROBLEM"

3.1 Model I

3.1.1 Definition

Consider the problem with

- a single commodity
- a single opportunity to replenish the inventory
- a demand which is random with known discrete probability distribution.

Further notations:

D = demand in units (each quantity has associated non-zero probabilities)
Q = order quantity
Sp = selling price
Cp = purchase cost (Cp < Sp)
Sv = salvage value (Sv < Cp) = selling price for any inventory remaining after demand has occurred
Cu = unit cost of under-ordering (shortage cost) Cu = Sp – Cp
Co = unit cost of over-ordering (overage cost) Co = Cp – Sv
p_x = probability distribution of demand

$$p_x \equiv P(D = x), \quad F(x) \equiv P(D \le x)$$

TC = total cost

$$TC(Q) = Co \cdot (Q - D); D \le Q,$$
$$= Cu \cdot (D - Q); D \ge Q$$

Total cost is a random variable with expected value:

$$\overline{TC}(Q) = E(TC(Q)) = \min_{Q \ge 0}$$

The inference of the optimal order (inventory level):

$$\overline{TC}(Q) = E(TC(Q)) =$$
$$= \sum_{x=0}^{Q} Co \cdot (Q - x) \cdot p_x + \sum_{x=Q}^{\infty} Cu \cdot (x - Q) \cdot p_x = \mathbf{min}; Q = 0,1,2,\dots$$

$$\overline{TC}(Q+1) = \sum_{x=0}^{Q+1} Co \cdot (Q+1-x) \cdot p_x + \sum_{x=Q+1}^{\infty} Cu \cdot (x - Q - 1) \cdot p_x$$

The difference between the previous formulas is:

$$\overline{TC}(Q+1)-\overline{TC}(Q)=Co\cdot\sum_{x=0}^{Q}p_x-Cu\cdot\sum_{x=Q+1}^{\infty}p_x=$$

$$=Co\cdot F(Q)-Cu\cdot(1-F(Q))\geq 0$$

$$\overline{TC}(Q)-\overline{TC}(Q-1)=Co\cdot F(Q-1)-Cu\cdot(1-F(Q-1))\leq 0$$

$$F(Q^*-1)\leq\frac{Cu}{Co+Cu}\leq F(Q^*)\qquad\qquad \frac{Cu}{Co+Cu}=\text{service level}$$

So the optimal order quantity Q* can be calculated as follows:

$$Q^*=F_{-1}\left(\frac{Cu}{Co+Cu}\right)$$

3.1.2 Example and calculation according to the inventory model I

Consider a retailer who sells Christmas trees. Before Christmas time he purchases spruces, each at a cost of 65 CZK per one meter of the tree. The selling price is 100 CZK per one spruce one meter high. If the stockout occurs, the retailer cannot reorder. In case of overstock, he can sell each spruce for 20 CZK per meter to a factory (for other processing - as a combustible wood). Based on historical data the probability distribution of demand is known (Table 1). The retailer would like to know how many spruces should be purchased.

Table 1

Demand	0	5	10	15	20	25	30	35
Probability	0,01	0,05	0,12	0,2	0,35	0,15	0,11	0,01
F(x)	0,01	0,06	0,18	0,38	0,43	0,88	0,99	1

$$Cu = Sp - Cp = 100 - 65 = 35\ CZK$$

$$Co = Cp - Sv = 65 - 20 = 45\ CZK$$

$$\frac{Cu}{Co+Cu}=\frac{35}{45+35}=0{,}4375\qquad P(x\leq 15)=0{,}38<0{,}4375<0{,}73=P(x\leq 20)$$

$$TC(15)=234{,}75\ CZK\qquad\qquad TC(20)=211{,}75\ CZK$$

Expected value of the total cost in case the retailer orders 20 spruces is lower in comparison with the case of the order of 15 spruces, so for minimal cost it is better to purchase 20 trees. Calculating the expected profit, the situation is the same (the expected profit for 15 spruces is 425 CZK, for 20 trees it is 448 CZK which is better).

3.1.3 Simulation of the inventory model I

Let's now try to solve the same example but using simulation methods. Simulation is the imitation of the operations of a real-world processes or systems and is mostly used for the description and analysis of the behaviour of a system. The base elements of the simulation are independent random numbers that are distributed continuously and uniformly between 0 and 1. In MS Excel we obtain these numbers using the function RANDOM()[1]. These numbers can be converted to the desired statistical distribution using various methods.

As it has been mentioned above, demand varies from 0 to 35 with given probabilities. To comply with it, let's create a table with 100 values where 0 is involved once, number 5 is there five times etc. (according to the probabilities). Then the demand is generated from this table by the function INDEX, which picks out given row and column from the table (or array). For the description of the row the function RANDOM() is used as follows:

=RANDOM() * 100 +1 [2]

That means that we want to find (randomly) any row between 1 and 100. Now the demand is generated.

The second step is to calculate profit or cost. Let's calculate the profit. We can use Cu and Co as before but now in Excel in the function IF. So the formula is:

=IF(Q<D;Q*Cu;D*Cu+(D-Q)*Co)

(Evidently instead of the names of the variables and constraints the references to the corresponding sells must be applied.)

Finally it is possible to generate the demand again and again (using the F9 key) and watch the changes of the profit. But the more elegant way is using Data Tables where the first row is constructed of various possible orders, first column contains attempt numbers and in the upper left corner the reference to the profit formula is entered. Via "Tools" and "Data table" this table is quickly filled by recalculated profits for the given order using generated demand. The results are in Table 2. As you can see, the best alternative according to the average profit is also 20 spruces. But it is not all. The table shows various possibilities that may happen, so it is possible to review the situation, and maybe take a different decision.

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1 In the Czech version of MS Excel this function is called NÁHCÍSLO()

2 In this case of row selection it is not necessary to generate integer values

Table 2

profit / orders	**700**	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**35**
number of test	1	0	175	350	525	700	-325	650	25
	2	0	175	350	525	700	875	-550	-375
	……	……	……	……	……	……	……	……	……
	499	0	175	350	525	700	75	250	-775
	500	0	175	350	525	-900	475	650	25
Average profit		0	168.6	320.4	428.2	448.8	277.4	135.6	-75.8
Min. profit		0	-225	-450	-675	-900	-1125	-1350	-1575
Max. profit		0	175	350	525	700	875	1050	1225

3.2 Model II

3.2.1 Definition

Consider the problem with

- a single commodity
- a single opportunity to replenish the inventory
- a demand which is normally distributed with known μ and σ

All the assumptions and notation are the same as before. Only for determination of the smallest order quantity that provides given service level, we have to use the following relationship: $Q^* = \mu + \sigma Z$

where Z is the standard normal probability distribution value that can be obtained in MS Excel using NORMSINV function with the parameter equal to the service level.

3.2.2 Example and calculations of the inventory model II

The florist purchases roses for 25 CZK per unit and sells them for 45 CZK per unit. If the roses are not sold during a day, the price has to be reduced to 20 CZK per unit for the next day. If there is no special occasion, the usual demand is 200 roses with standard deviation of 50 ones. How many roses should be ordered?

$$Cu = Sp - Cp = 45 - 25 = 20 \text{ CZK}$$

$$Co = Cp - Sv = 25 - 20 = 5 \text{ CZK}$$

$$\frac{Cu}{Co + Cu} = \frac{20}{5 + 20} = 0{,}8 \qquad \text{NORMSINV}(0{,}8) = 0{,}841621$$

$$Q^* = \mu + \sigma Z = 200 + 50 * 0{,}841621 = 242{,}08$$

The optimal order according to the given data is 242 roses.

3.2.3. Simulations of the inventory model II

Now it is necessary to generate a demand that follows normal distribution with given mean and standard deviation. It is done by the function NORMINV(probability;mean,;std.dev), where the function RANDOM() is used as the probability, so the formula in MS Excel is:

=NORMINV (RANDOM();200;50)

Again it is better and quicker to use Data Table. The only problem is how to set the orders. After few simulations it was clear that the interval between 230 and 270 units is sufficient (and the difference of 5 units between two orders is irrelevant). But now the simulation does not fully confirm the results of the mathematical model. The average profit ranges between 3100 CZK and 4100 CZK but it is not possible to determine the optimal order. In Chart 1 the average profits for ten different simulations and for the given orders are displayed. The order of 240 or 245 roses was the best only in 3 cases. Even in one simulation it seemed that it would be the worst choice because of the profit of only 50 CZK. So in situations like this the description of the demand distribution should be more accurate to obtain more reliable results.

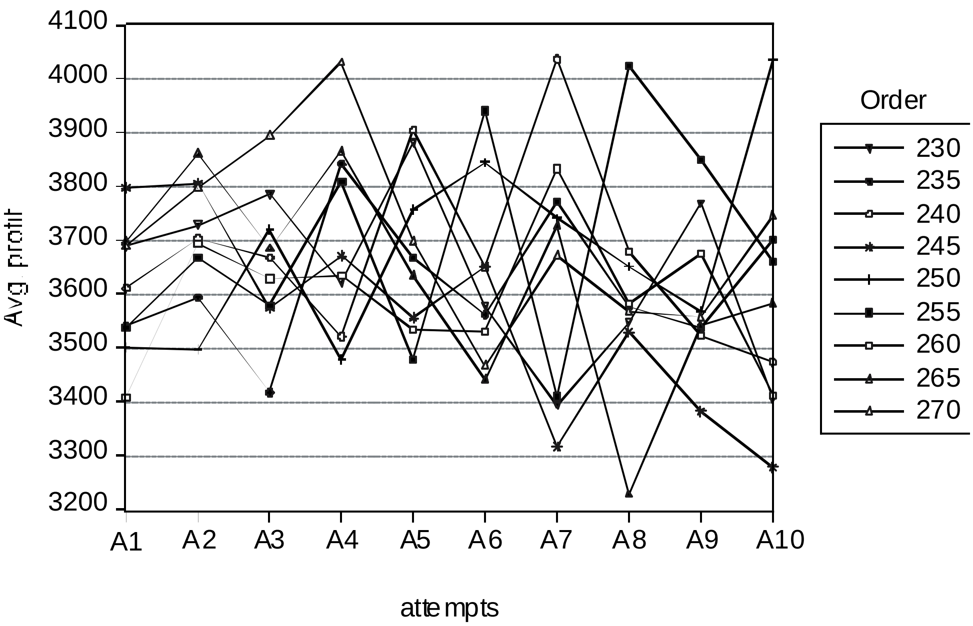

Chart 1 - Average profits for various orders in 10 simulations

4. CONCLUSION

The two conceptions described above are both available for the situations where only one order in one time period is possible. The analytical approach is more suitable for the case with known discrete probability distribution of demand (model I) but still it is necessary to know the definition of the mathematical model and the enumeration. On the other hand, simulation is advisable for more complicated situations. It is necessary to obtain a lot of information and to define demand distribution precisely. Easier solution in a spreadsheet (having no knowledge of the mathematical methods) can be the advantage of the simulation as well as no additional investment in the special software.

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DEBT OVERHANG AND ECONOMIC GROWTH: AN ECONOMETRIC EXPLORATION

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Abstract This paper looks at whether external debt and capital flight could be potential explanations for growth rate differences across the developing world. Although there is a wide-ranging of theoretical literature on this issue, there are only few empirical studies that show that there is an inverse relationship between growth and external imbalances. The critical innovation of this paper is the empirical exploration on the impacts of external debt on growth once total debt stock is decomposed according to source and maturity structures.

1. INTRODUCTION

It is now apparent that least developing countries (LDCs) in general and Sub-Saharan African countries (SSA) in particular have been marginalized and bypassed by the process of globalization. The deterioration in income per capita growth for SSA is remarkable. The growth in per capita for SSA was worsening from decade to decade, and becoming worst in the last two decades. The growth in per capita income that was around 1.9% in the 1960s declined to around 1.1% in the 1970s, to 0.05% in the 1980s and to 0.04% in the 1990s.The figures turned even worse when one takes a longer time horizon. The growth rate of per capita income during the periods (1970-2000) and (1980-2000) were indeed negative. This is in contrast to a 2.8% growth for the whole world in the 1960s, 2.2% in the 1970s, to 1% in the 1980s, and to 1.3% in the 1990s. Moreover, many empirical studies show the phenomenon of divergence in real per capita growth across the world economy at large, hence the poor getting poorer while the rich getting richer, eventually increasing the dispersion of income per capita across countries and over time. Such empirical studies also point out that the degree of divergence in real income per capita was worse in the 1980s and 1990s as opposed to the 1960s and partly the 1970s. This very question has in fact drawn the attention of many economists across the globe. The poor performance (misperformance) of Sub-Saharan Africa or "Africa's growth tragedy" as Easterly (2000) correctly puts it, has been explained from various fronts. The potential factors range from bad policies and external shocks (Hadjimichael, et al, 1995; and Rodrik, 1999, among others), to ethnic fractionilization (Easterly, 2000, among others), to gender inequality in education (Klasen, 2002), and to geographic location (Sachs, et al 1998), among others.

Therefore, in this paper, I argue that though the issues of policy, education, geography and ethnicity are important determinants of long-term growth, they fail to provide full explanation of why growth rate differences across countries were so dramatic in the 1980s and 1990s as compared to the decades earlier. In this context, there is a wide spread consensus that the 1980s have been considered by many as the "lost decade" for Africa and Latin America in terms of growth and development. Singer (1990, in: Healey, 1995), for example, expresses Sub-Saharan Africa as the region that is converging to acquire the character of a marginalized 'Fourth World', increasingly recognized as requiring special action and criteria. In the same token, the 1980s had been far from favourable for Latin American countries.[1] The prime suspects in this regard are external imbalances that in fact become cardinal issues in the entire developing world notably those in SSA. This issue becomes even more aparent given the truth that 33 of the 41 countries characterized by the World Bank and IMF as heavily indebted poor countries (HIPCs) are located in Africa. This paper, therefore, will rather focus on the extent to which external imbalances have accounted for growth-rate differences across the developing world in the past two decades.

2. THE EXTERNAL IMBALANCES-GROWTH NEXUS: *A REVIEW OF THEORETICAL LITERATURE*

In general, it is quite clear that LDCs have strong incentives to borrow overseas in order to finance their domestic investment, which is indispensable to achieve economic growth.[2] Part of the reason is that since poor countries are far away from their steady states, any investment injection could lead them to have accelerated economic growth. Though this is generally true, it turns out that capital inflows in the form of external debt to LDCs enhance growth only to a certain limit. Once debt gets bigger and becomes unmanageable (unsustainable), it rather becomes a major destabilizing factor and a serious bottleneck to long-run economic growth. Moreover, as long as scarce resources are not wisely invested in projects that have the expected returns higher than the cost of foreign debt, they may endanger the long term growth prospect of the country under consideration and leads to low economic growth, higher demand for external debt and more external imbalances.

The external debt-growth literature points out various mechanisms through which debt is translated into sluggish economic growth. One of the channels in this

1 "Not long ago, Latin American countries seemed to be condemned to a life of despair. During the 1980s, after the onset of the debt crisis, growth rates, which during the 1970s oscillated around 6% collapsed to an average 1.8%. From the perspectives of the 1980s, even future growth prospects were clouded by a sharp drop in the share of capital formation from about 20% in the 1970s to about 16% in the years following 1982" (WEO, Oct. 1993, in: Kaminsky, et al. 1996, p. 1)

2 As many argue, the USA in the 19 Century, The Marshall Plan for a war-torn Germany and East Asian countries in recent periods are all cases in point where foreign capital was translated into long-run economic growth.

respect is the so-called the debt overhang hypothesis. This theory suggests that once it becomes apparent that there is a real threat that the future total debt stock of a country will be much larger than the country's repayment potential, the expected debt service will be an increasing function of the country's output level (Pattillo, et al. 2001, Claessens, 1996, among others).

As the result the expected rate of returns from productive investments in such an economy will be low as the significant portion of any subsequent economic progress will be eaten up by creditors, which further reduces both domestic and foreign investments and eventually downsizes economic growth (Krugman, 1988, Sachs, 1989a, among others).

The premise that debt to a certain limit, if wisely utilized and properly managed, plays a pivotal role in enhancing long run economic growth but retards it if its level is increasing over time is generally linked to the so-called the debt laffer curve.[3] The debt overhang problem is linked to the transfer of resources from capital scarce to capital surplus countries. In this respect, Krugman (1988) defined debt overhang as "the presence of an existing inherited debt sufficiently large that creditors do not expect with confidence to be fully repaid" (p.254).

Claessens and Diwan (1990) argue that "debt overhang is a situation in which the illiquidity effect, the disincentive effect, or both effects are strong enough to discourage growth in the absence of concessions by creditors" (p. 31). This is also known as a "narrow" definition of the debt overhang where the impact of a high external debt that is linked to the tax disincentives argument, where any success in indebted country's economic performance is taxed away by creditors and ultimately little is left over for domestic investment and subsequent growth (Hjertholm, 2001).

2. 1. A FORMAL THEORETICAL MODEL OF THE DEBT OVERHANG HYPOTHESIS

Most theoretical models of the debt overhang hypothesis start from the same assumption (a two period model) where a country is assumed to have carried over from the previous period some amount of external dent, D, which must be paid in the final period.[4]

Repayment(R) is given by:

$$R = \min\left(D, Y - \overline{C}\right)$$

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3 The debt laffer curve argument (which was apparently introduced by Jeffrey Sachs) is derived from the tax laffer curve hypothesis introduced by Arthur Laffer (1981), who argues that if personal tax rates were raised, they generate a dreadful impact on government tax revenue. The reason is that high tax rates either simply discourages investment or leads to tax evasion.

4 This paper follows the debt overhang hypothesis model by Borensztein (1990). Similar approaches may be found in Agenor and Montiel (1996), and Krugman (1988), among others.

where,

D – initial amount of debt,

Y – output, and

$\overline{C}$ – fixed amount of output that the debtor country can always keep for consumption.

The model also assumes two states of nature: a favorable state (G) and an unfavorable one (B), where productivity is expected to be higher in the former and lower in the later states of nature, respectively. Assuming that Y is a function of investment carried out in the first period, given the state of nature: $Y^{s} = \theta^{S} f\left(I\right)$, where, s stands either for G or B alternatively. The model also assumes that there is a physical upper limit to investment $\overline{I}$, and that there is no overlap of output as the two states are so different, which implies that $\theta^{B} f\left(\overline{I}\right) \prec \theta^{G} f\left(0\right)$.

Then, a country experiences a debt overhang if $D \succ Y^{B} - \overline{C}$ because there are not enough resources to be surrendered for fully servicing the contractual debt obligation in a bad state (B). The debt overhang, therefore, creates a disincentive effect on domestic investment. If we have the situation whereby $Y^{B} - \overline{C} \prec D \prec Y^{G} - \overline{C}$, then debt is serviced in full during favorable time. In contrast, during unfavorable time, debt is serviced only the amount that is left over from the available output after the fixed consumption is taken out.

Following this model, for the debtor country, the marginal return that it receives for every additional investment is $p\theta^{G} f_{I}$ where p is the probability of the occurrence of favorable situation for higher productivity. In contrast, if debt overhang was not at the stake, the expected marginal return out of every unit of investment would be substantially larger: $p\theta^{G} f_{I} + \left(1 - p\right)\theta^{B} f_{I}$.

The model also suggests that if $D \succ Y^{G} - \overline{C}$, the return to investment would be zero and therefore, a debtor country does not have any reason whatsoever to commit additional resources for boosting investment.[5]

2. 2. THE EMPIRICAL MODEL AND DATA DESCRIPTION

$$\ln\left(y\right) - \ln\left(y_{0}\right) = \left(1 - e^{\lambda t}\right)\left[\begin{array}{l} -\left(\dfrac{\xi}{1-\xi}\right)\ln\left(n + g + \delta\right) + \left(\dfrac{\alpha}{1-\xi}\right)\ln\left(S_{p}\right) \\ +\left(\dfrac{\beta}{1-\xi}\right)\ln\left(S_{h}\right) + x\theta - \ln\left(y_{0}\right) + gt + \ln\left(A_{0}\right) \end{array}\right]$$

where, T is the length of time under consideration.

5 This is consistent with all the empirical and theoretical literatures on debt overhang (Deshpande, 1997), Claessens, et, al.(1996), Elbadawi, et, al. (1997), among others.

3. REGRESSION RESULTS

Two kinds of strategies are employed to empirically investigate the issue of external debt’s accountability on growth. The first one is the fixed effects (FE) versus the random effects (RE) approach. To distinguish which of the two models is more appropriate, the Hausman test is used. In all the regressions, except the capital flight, and debt variables, other covariates that always appear in the augmented Solow growth framework are added.

Now, turning to the results themselves, they indicate that past-accumulated debt is negatively related to growth of real GDP per capita; suggesting the existence of a debt overhang phenomenon across developing countries, controlling for other variables, though the significance disappears once export growth is excluded. In contrast to other studies, this paper does not show a significant (though a positive) relation between current debt ratio and growth of real GDP per capita. Similarly, like in other studies, I have not found a statistically significant negative relationship between total debt service ratio and growth of real GDP per capita, though this variable always bear the expected sign. I, therefore, alternatively use interest payments as a proxy for the actual cost of foreign debt, though these neither have been statistically significant.

Turning to the main focus of this paper, i.e., the regression results, once total external debt stock is decomposed, suggest several things. From the regression results, it implies that while the initial short term debt to export ratio (STDXI) component of the total debt stock (TED) negatively impacts on growth, the initial long term debt to exports ratio (LTDXI) component of TED induces economic growth. This is true both in the RE and FE models, despite the insignificance of LTDXI in the later once the export growth is included. Other results seem to indicate that debts that were channeled to the private sector enduce economic growth, while the part of the external debt used to inject the public sector punishes economic growth. The results of the regression once total debt stock is decomposed according to concessionality indicate that while the non-concessional component of the external debt punishes economic growth (through a debt overhang and or liquidity effects), the concessional debt remains insignificant, though have the expected sign in the RE, but a wrong sign in the FE model. Other results indicate that borrowing from the International Bank for Reconstruction and Development to exports ratio (IBRDXI), is included to capture the impact of market based loans, loans from International Development Associationto exports ratio (IDAXI) is used as proxy for foreign aid and partly concessional lending and loans from IMF to exports ratio (IMFXI) is included to capture loans in exchange for policy (which may also be an indirect measure of the impact of the structural adjustment program).The results indicate that while both loans from the World Bank and IDA induce growth (the latter being highly significant in both RE and FE models, though it is insignificant in the FE model, when export growth is added), loans from the IMF retards growth probably indicating the failure of the IMF adjustment program in most developing countries, notably those of Sub-Saharan Africa. The last but not least attempt was to decompose total external debt into bilateral and multilateral sources to figure out whether the kind of creditors matters for growth. The regression results suggests that while BLATX has failed to contribute positively to growth, having either a negative or insignificant signs, loans from multilateral sources have a positive and statistically significant impact on growth, despite its insignificance in the FE model once export is removed from the equation. In contrast, MLATX should promote growth as it to a large extent is policy or poverty driven and often is accompanied by low interest rates.[6]

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[6] For the broader discussion on the impact of aid on economic growth, see, Burside and Dollar (2000) and Hansen and Tarp (2001).

Table (1): The impact of past and current total external debt on growth of real GDP per capita of (controlling for other variables) (1982-99)

	Random effects model				Fixed effects model			
Variable	1	2	3	4	5	6	7	8
CONST	2.987 (1.05)	6.619** (2.02)	2.645 (0.72)	8.798** (2.02)	5.822* (1.78)	50.23*** (4.83)	67.53*** (5.89)	49.88*** (4.73)
LGDPI	-0.692* (-1.87)	-1.023*** (-2.58)	-0.811* (-1.68)	-1.361*** (-2.66)	-0.923** (-2.31)	-7.127 (-5.33)	-9.880*** (-6.87)	-6.963*** (-5.08)
CFLGDPI	-0.016 (-1.29)	-0.014 (-1.12)	-0.006 (-0.39)	-0.003 (-0.20)	-0.012 (-0.99)	-0.013 (-1.01)	-0.011 (-0.69)	-0.011 (-0.84)
LEXPG	7.309*** (9.87)	7.086*** (9.59)			6.994*** (9.42)	4.968**** (5.80)		5.054*** (5.85)
TEDX2	-5.51e-07** (-1.99)	-5.12e-07* (-1.87)	-9.95e-08 (-0.29)	-5.36e-08 (-0.16)	-5.18e-07* (-1.90)	-5.68e-07* (-1.91)	-1.64e-07 (-0.44)	-6.61e-07** (-2.00)
TEDX	0.001 (1.13)	0.01 (1.38)	0.0005 (0.51)	0.0009 (0.86)	0.001 (1.46)	0.002 (1.33)	0.002 (1.26)	0.001 (0.91)
TDSX	-0.025 (-1.40)	-0.192 (-1.05)	-0.034 (-1.45)	-0.023 (-1.02)		-0.065* (-1.91)	-0.094** (-2.44)	
INTX					-0.036 (-1.44)			-0.052 (-1.30)
GGC	-0.021 (-0.65)	-0.019 (-0.58)	-0.095** (-2.26)	-0.087** (-2.11)	-0.026 (-0.79)	-0.091 (-1.50)	-0.099 (-1.42)	-0.095 (-1.74)
LGCF	2.145*** (3.64)	1.891*** (3.17)	3.395*** (4.57)	2.916*** (3.90)	1.940*** (3.31)	2.195** (2.19)	3.502*** (3.11)	2.179** (2.15)
CPI	-0.001 (-1.45)	-0.001 (-1.50)	-0.001 (-1.07)	-0.001 (-1.14)	-0.001 (-1.54)	-0.001 (-1.19)	-0.001 (-0.83)	-0.0009 (-0.94)
TOTG	0.026*** (2.83)	0.027*** (2.92)	0.006 (0.55)	0.008 (0.71)	0.027*** (2.94)	0.208** (2.07)	0.005 (0.52)	0.022** (2.19)
POPG	-1.475*** (-3.17)	-1.328*** (-2.84)	-1.626*** (-2.72)	-1.367*** (-2.30)	-1.291*** (-2.78)	-0.678 (-0.93)	0.128 (0.16)	-0.739 (-1.01)
LFG	1.212** (2.46)	1.142** (2.33)	1.612*** (2.57)	1.465** (2.38)	1.125** (2.31)	0.0461 (0.73)	0.060 (0.08)	0.480 (0.75)
SCHL	0.151 (0.38)	0.073 (0.19)	-0.0005 (-0.00)	-0.135 (-0.26)	0.064 (0.17)	1.112 (0.92)	1.121 (0.81)	0.664 (0.56)
VIOL	-0.298 (-0.62)	-0.510 (-1.05)	-0.239 (-0.38)	-0.579 (-0.93)	-0.527 (-1.11)	-1.446* (-1.82)	-1.697* (-1.85)	-1.432* (-1.78)
PD2	-1.025 (-1.30)	-1.003 (-1.28)	-0.632 (-0.63)	-0.624 (-0.64)	-1.091 (-1.40)	-0.372 (-0.39)	-0.178 (-0.16)	-0.217 (-0.23)

Variable	1	2	3	4	5	6	7	8
PD3	-3.237*** (-6.98)	-2.995*** (-6.35)	-3.363*** (-5.75)	-2.945*** (-4.99)	-3.118*** (-6.42)	-0.591 (-0.67)	0.655 (0.66)	-0.437 (-0.48)
HIPC		-1.264** (-2.14)		-2.152*** (-2.83)	-1.200** (-2.06)			
N	175	175	175	175	175	175		175
R^2	0.58	0.60	0.33	0.38	0.60	0.69	0.59	0.69
Chi^2	0.000	0.000	0.000	0.000	0.000			

The num bers in pa ran the ses are t-Statistics (two-tailed).

*. Sig nif i cance at 10% level.

**. Sig nif i cance at 5% level.

***. Sig nif i cance at 1% level.

De pen dent va riab le is: log of the growth rate of GDP per ca pi ta, and this ap pli cab le for all tab les.

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ANALYSIS OF STYLE INVESTING OF THE MUTUAL FUNDS IN SLOVAKIA

ZDENKA MILÁNOVÁ

Abstract We try to study prices of asset, because some investors classify risky assets into different styles and move funds between these styles depending on their relative performance. We assume that news about one style can affect the prices of other apparently unrelated style, that assets in the same style commove to much while assets in different styles commove too little and high average returns on a style will be associated with common factors for reason unrelated to risk. In the last part we try to apply this conclusions on Slovak financial market.

Keywords: Mutual funds, Style Investing, Style Analysis

Classification is the grouping of objects into categories based on some similarity among them [8]. Classification of large number of objects into categories is also in financial markets. The asset classes that investors use in this process are sometimes called investing styles, and the process of allocating funds among styles is known as style investing.

There are several reasons why investors use investing styles:

1. Categorization simplifies problems of choice and allows us to process vast amounts of information reasonably efficiently.
2. Creation of asset categories helps investors evaluate the performance of professional money managers, because style automatically creates a group of managers who pursue that particular style.
3. Style investing simplifies the process of diversification.

Let's create a model of style investing. We consider an economy with $2n$ risky assets in fixed supply, and a risk free asset, cash, in perfectly elastic supply and with zero net return. We model risky asset i as a claim to a single liquidating dividend $D_{i,T}$ to be paid at some later time T. The eventual dividend equals

$$D_{i,T} = D_{i,0} + \varepsilon_{i,1} + \ldots + \varepsilon_{i,T}, \quad (1)$$

where $\varepsilon_{i,t}$ becomes known at time t [5]. We assume

$$\varepsilon_t = \left(\varepsilon_{1,t}, \ldots, \varepsilon_{2n,t}\right)' \sim N\left(0, C_D\right).$$

The price of a share of risky asset l at the time t is $P_{l,t}$ and the return on the asset between time t–1 and time t is

$$\Delta P_{l,t} = P_{l,t} - P_{l,t-1}. \tag{2}$$

We build a simple model of style investing. There are just two styles, X and Y, and each risky asset in the economy belongs to one, and only one, of these two styles Risky asset 1 trough n are in style X, while $n+1$ through $2n$ are in style Y. For now, we assume that the composition of two styles is the same in every time period.

As a measure of the value of style X in time t, we use $P_{X,t}$, defined as the average price of a share across all assets in style X:

$$P_{X,t} = \frac{1}{n}\sum_{l\in X} P_{l,t}. \tag{3}$$

The return on style X between time t–1 and time t is

$$\Delta P_{X,t} = P_{Xt} - P_{X,t-1} \tag{4}$$

There are two kinds of investors in our model, "switchers" and "fundamental traders". The investment policy of switchers has two distinctive characteristics:

1. switchers allocate funds at the level of a style
2. how much they allocate to each style depends on that style's past performance relative to other style.

Each period, switchers allocate more funds to styles with better than average performance and finance these additional investments by taking funds away from styles with below average performance.

Switcher's demand [1] for shares of an asset i in style X at time t, is:

$$N_{i,t}^{S} = \frac{N_{X,t}^{S}}{n} = \frac{1}{n}\left[A_X + \sum_{k=1}^{t-1}\theta^{k-1}\left(\frac{\Delta P_{X,t-k} - \Delta P_{Y,t-k}}{2}\right)\right], \text{ pre } i = 1, 2, ..., n, \tag{5}$$

where A_X and are constants, with $0<\theta<1$.

The second investor type in our model is fundamental trader. They act as arbitrageurs and try to prevent the price of an asset from deviating too far from its expected final dividend. Since they have no constraints on their allocations, they solve

$$\max_{N_t} E_t^F\left(-\exp\left[-\gamma\left(W^F + N_t'\left(P_{t+1} - P_t\right)\right)\right]\right), \tag{6}$$

where $N_t = \left(N_{1,t},\ldots,N_{2n,t}\right)'$ is a vector of the number of shares allocated to each risky asset, γ governs the degree of risk aversion, E_t^F denotes fundamental traders expectations at time t and $P_t = \left(P_{1,t},\ldots,P_{2n,t}\right)'$. Optimal holdings N_t^F are given:

$$N_t^F = \frac{\left(V_t^F\right)^{-1}}{\gamma}\left(E_t^F\left(P_{t+1}\right) - P_t\right), \tag{7}$$

where

$$V_t^F = \mathrm{var}_t^F \left(P_{t+1} - P_t \right),$$

with the F superscript in V_t^F again denoting a forecast made by fundamental traders.

An individual stock-level momentum strategy ranks all stocks on their return in the previous period and buys those stocks that did better than average and sells those that did worse. It can by implemented through

$$N_{i,t} = \frac{1}{2n}\left[\Delta P_{i,t} - \Delta P_{M,t}\right], \quad i = 1, ..., 2n, \tag{8}$$

where

$$\Delta P_{M,t} = \frac{1}{2n}\sum_{l=1}^{2n} \Delta P_{l,t}.$$

An individual stock-level value strategy buys (sells) those stocks which are trading below (above) fundamental value:

$$N_{i,t} = \frac{1}{2n}\left[P_{i,t}^{*} - P_{i,t}\right], \quad i = 1, ..., 2n. \tag{9}$$

A style-level momentum strategy buys into styles with good recent performance and avoids styles that have done poorly:

$$N_{i,t} = \frac{1}{2n}\left[\frac{\Delta P_{X,t} - \Delta P_{Y,t}}{2}\right], i \in X, \tag{10}$$

$$N_{j,t} = \frac{1}{2n}\left[\frac{\Delta P_{Y,t} - \Delta P_{X,t}}{2}\right], j \in Y.$$

A **style-level value strategy** buys into styles trading below fundamental value and shorts the remaining styles

$$N_{i,t} = \frac{1}{2n}\left[P_{X,t}^{*} - P_{X,t}\right], i \in X, \tag{11}$$

$$N_{j,t} = \frac{1}{2n}\left[P_{Y,t}^{*} - P_{Y,t}\right], j \in Y.$$

Essentially, in **return based style analysis** a factor model is used to explain fund returns. Return based analysis determines **the mimicking portfolio of mutual funds** or other investment opportunities with positive portfolios weights, i.e., the positively weighted style portfolio that is closest to the mutual fund in a least squares sense.

The case when no constraints are imposed on the factor loadings will be referred to as weak style analysis. The case where only the portfolio constraints is imposed will be referred to as semi–strong style analysis and the case where both portfolio and the positivity constraints are imposed will be referred to as strong style analysis [4].

Suppose that K factor (mimicking) portfolios with return vector R_t drive the asset returns. In addition, there are N mutual funds with return vector r_t, for which we have the linear factor model

$$r_t = a + BR_t + \varepsilon_t, \tag{12}$$

where $E[\varepsilon_t] = E[\varepsilon_t R_{i,t}] = 0$ for $i = 1, \ldots, K$.

In this case $B = C_{rR} C_{RR}^{-1}$ and $a = \mu_r - B\mu_R$, where C_{rR} is a covariance matrix between returns of mutual funds and mimicking, C_{RR} is a covariance matrix of returns of mimicking portfolio and μ is an expected return vector. When using (12) as a factor model, we do not impose any constraints on a and B. If there are no restriction on B, we refer to this as weak style analysis.

If we define a_i as the ith element of a and b_i as the ith row of B, then a_i and b_i are the solutions to the problem

$$\min_{\alpha,\beta} E\left[\left(r_{i,t} - \alpha - \beta' R_t\right)^2\right]. \tag{13}$$

The vector b reflect the fund mimicking positions or the minimum variance hedge position for the mutual fund.

To see the effect of the portfolio constraint $\sum_j \beta_j = 1$, let $\tilde{a}_i$ and $\tilde{b}_i$ be the solutions of the problem

$$\min_{\alpha,\beta}\left[\left(r_{i,t} - \alpha - \beta' R_t\right)^2\right], \tag{14}$$

s.t.

$$\beta' e_K = 1,$$

where e_K is K-dimensional vector of ones. Thus, $\tilde{b}_i$ are the factor exposures which are constrained to sum to one, i.e., they characterize a portfolio. The case where only the portfolio constraints is imposed, will be referred to as semi-strong style analysis. Using standard least squares results, it is straightforward to show that the coefficients $\tilde{b}_i$ can be written as

$$\tilde{b}_i = b_i + \left(1 - b_i' e_K\right) C_{RR}^{-1} e_K \left(e_K' C_{RR}^{-1} e_K\right)^{-1}. \tag{15}$$

In addition to the portfolio constraints, it is common in style analysis to impose positivity constraints on the estimated factor exposures. The style portfolios $\hat{b}_i$ and the associated intercepts $\hat{a}_i$ are then the solutions to the problem

$$\min_{\alpha,\beta} E\left[\left(r_{i,t} - \alpha - \beta' R_t\right)^2\right], \tag{16}$$

s.t.

$$\beta' e_K = 1,$$
$$\beta \geq 0.$$

We refer to this case as strong style analysis.

The strong style coefficients as given in (16) reflect the positively weighted portfolio of the benchmarks that mimics the mutual fund.

Because we have no detailed information about portfolio of mutual funds in Slovakia, to see consequences of propositions in this article, we are going to create two fund of funds. The first one consists of three obligation mutual funds and the second one consists of three stock mutual funds. The first one we call investing style X and the second one investing style Y. In investing style X are these three funds: TAM - Korunový dlhopisový, VÚBAM - Eurofond a VÚBAM - Korunový. Investing style Y consists of these funds: TAM - Americký akciový, JTAM - Japan akciový a JTAM - Euro akciový. We have weekly data from 1. 1. 2002 to 31. 1. 2003. To get more information see [10].

Table 1: *Individual stock-level momentum strategy in investing style X*

Week	TAM - Korunový		VÚBAM - Eurofond		VÚBAM - Korunový		$\Delta P_{M,t}$
t	Return	$N_{1,t}$	Return	$N_{2,t}$	Return	$N_{3,t}$	
1	0,38%	0,03%	0,47%	0,04%	0,38%	0,03%	0,21%
2	0,34%	0,11%	-0,58%	-0,04%	0,35%	0,12%	-0,34%

Table 2: *Individual stock-level momentum strategy of investing style Y*

Week	TAM - Americký		JTAM - Japan		JTAM - Euro		$\Delta P_{M,t}$
t	Return	$N_{4,t}$	Return	$N_{5,t}$	Return	$N_{6,t}$	
1	0,06%	-0,03%	0,00%	-0,04%	0,00%	-0,04%	0,21%
2	-2,11%	-0,30%	0,07%	0,07%	-0,12%	0,04%	-0,34%

***Table 3:** Individual stock-level value strategy of investing style X*

Week	TAM - Korunový		VÚBAM - Eurofond		VÚBAM - Korunový	
t	NAV	$N_{1,t}$	NAV	$N_{2,t}$	NAV	$N_{3,t}$
1	1,5713	1,21%	1,0444	-0,41%	1,0859	0,56%
2	1,5772	1,11%	1,0493	-0,49%	1,0900	0,50%
	$P^*_{1,t}$	1,6439	$P^*_{2,t}$	1,0197	$P^*_{3,t}$	1,1197

***Table 4:** Individual stock-level value strategy of investing style Y*

Week	TAM - Americký		JTAM - Japan		JTAM - Euro	
t	NAV	$N_{4,t}$	NAV	$N_{5,t}$	NAV	$N_{6,t}$
1	0,9734	-2,39%	0,9996	-1,99%	0,9896	-2,45%
2	0,9740	-2,40%	0,9996	-1,99%	0,9896	-2,45%
	$P^*_{4,t}$	0,8298	$P^*_{5,t}$	0,8802	$P^*_{6,t}$	0,8424

***Table 5:** Style-level momentum strategy of investing style X and Y*

Week	Return of style X	Demand of X	Return of style Y	Demand of Y
t	$\Delta P_{X,t}$	$N_{i,t}$	$\Delta P_{Y,t}$	$N_{j,t}$
1	0,40%	0,03%	0,02%	-0,03%
2	0,08%	0,07%	-0,71%	-0,07%

***Table 6:** Style-level value strategy of investing style X and Y*

Week	Investing style X		Investing style Y	
t	NAV of style X	$N_{i,t}$	NAV of style Y	$N_{j,t}$
1	1,2339	0,45%	0,9875	-2,28%
2	1,2388	0,37%	0,9877	-2,28%
	$P^*_{X,t}$	1,2611	$P^*_{Y,t}$	0,8508

In the next two tables are results of week and strong style analysis. We try to create mimicking portfolio, which is combination of fund of funds VB - Quartett "Sicherheit" and index SAX. The reason is, that passively directed mutual funds have lower costs than actively directed mutual funds.

Table 7: *Week analysis of investing style*

Mutual fund	Intercept	b_1	b_2
Sporo - Korunový penažný	0,0013	-0,0075	-0,0790
TAM - Korunový penažný	0,0013	-0,0098	0,0114
VÚBAM - Penažný korunový	0,0010	-0,0079	-0,0817
Sporo - Korunový dlhopisový	0,0022	-0,0030	-0,0378
TAM - Dolárový dlhopisový	-0,0013	0,0431	0,5664
TAM - Euro dlhopisový	0,0006	0,0457	-0,1074
TAM - Korunový dlhopisový	0,0024	-0,0106	0,0695
VÚBAM - Dolárový	-0,0018	0,0856	0,7881
VÚBAM - Eurofond	0,0004	0,0852	0,4641
VÚBAM - Korunový	0,0022	-0,0101	0,2622
TAM - Americký akciový	-0,0052	0,1017	1,2246
TAM - Európsky akciový	-0,0067	0,1003	0,9541
TAM - Európsky technologický	-0,0082	0,1822	1,7973
VÚBAM - Svetové akcie	-0,0059	0,1486	1,6592
TAM - Korunový akciovo-dlhopisový	0,0017	0,1542	0,0667
TAM - Medzinárodný akciovo-dlhopisový	0,0017	0,1542	0,0667
VÚBAM - Vyvážený rastový	-0,0027	0,0927	0,2002

Table 8: *Strong analysis of investing style*

Mutual fund	Intercept	$\hat{a}$	$\hat{b}_1$	$\hat{b}_2$
Sporo - Korunový penažný	0,0017	0,0021	0,1556	0,8444
TAM - Korunový penažný	0,0016	0,0020	0,1401	0,8599
VÚBAM - Penažný korunový	0,0014	0,0018	0,1556	0,8444
Sporo - Korunový dlhopisový	0,0026	0,0030	0,1532	0,8468
TAM - Dolárový dlhopisový	-0,0012	-0,0010	0,1017	0,8983
TAM - Euro dlhopisový	0,0010	0,0014	0,2050	0,7950
TAM - Korunový dlhopisový	0,0019	0,0031	0,1307	0,8693
VÚBAM - Dolárový	-0,0018	-0,0017	0,1045	0,8955

Mutual fund	Intercept	$\hat{a}$	$\hat{b}_1$	$\hat{b}_2$
VÚBAM - Eurofond	0,0006	0,0008	0,1529	0,8471
VÚBAM - Korunový	0,0024	0,0027	0,1022	0,8978
TAM - Americký akciový	-0,0053	-0,0054	0,0527	0,9473
TAM - Európsky akciový	-0,0068	-0,0068	0,0921	0,9079
TAM - Európsky technologický	-0,0085	-0,0089	0,0352	0,9648
VÚBAM - Svetové akcie	-0,0062	-0,0065	0,0273	0,9727
TAM - Korunový akciovo-dlhopisový	0,0020	0,0022	0,2711	0,7289
TAM - Medzinárodný akciovo-dlhopisový	0,0008	-0,0001	0,1092	0,8908
VÚBAM - Vyvážený rastový	-0,0024	-0,0022	0,1989	0,8011

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OPENED MUTUAL FUNDS OUTRANKING FROM A VIEW POINT OF AN SLOVAK INVESTOR

VLADIMÍR MLYNAROVIČ

Opened mutual funds tend to become the most current form of investments in Slovak Republic as well. The present supply of investments opportunities from the site of the Association of Asset Management Companies (ASS) corresponds to the fact. For example, in information provided by ASS on April 17, 2003 one can find weekly data about 18 money market funds, 34 bond market funds, 54 equities market funds, 15 funds of funds and 17 mixed funds. The information is divided into three groups, namely there are a basic information, a overview of performance and a overview sales there. Among the basic information, besides the name of the fund, one can find the following data:

- currency of the share,
- value of the share,
- the change or the share value over the last week in %,
- paid gross dividends at the last week,
- the sum of shares in SKK that were given back over the last week,
- the sum of issued shares in SKK over the last week,
- maximum purchasing price of the share,
- minimum buying price of the share.

Among characteristics that describe performance of funds there are:

- minimum value of he first investment,
- maximum charge when a share is given back in %,
- maximum issued charge in %,
- maximum charge for asset management in % p.a.,
- performance of the fund for one month in %,
- performance of the fund for three months in %,
- performance of the fund for six month in %,
- performance of the fund for one year in %,
- performance of the fund for three years in % p.a..
- cumulated sum of net dividends per one share

The overview of sales consists of the following items about net sales in the Slovak Republic in Slovak crowns:

- for one week,
- for one month,
- for six month,
- for one year,
- cumulated sales.

It is clear that a potential investor faces in making investment decisions two problems. The first follows from relatively wide supply of investment opportunities and the second form a great amount of disposable information about fund performance in a close but also in a further past. Rational investor hardly can choose only a one characteristic as a criterion for investment decisions. However, if he chooses more of characteristics, in general it is nearly excluded that these various criteria of investment decision making will choose the same investment opportunity. The problem arises how to resolve a conflict of various criteria and to outrank funds "from the best to the worst" when all criteria are assumed simultaneously.

1. METHODOLOGY OF OPENED MUTUAL FUND OUTRANKING ILLUSTRATION

The problem of investment fund outranking can be formally written in the form of following multiple criteria decision making problem

$$\text{"max"}\left\{ y = \left(y_1, y_2, \ldots, y_k\right) \mid \ y \in Y \right\},$$

where elements of the set Y are assumed investments funds which of them is evaluated on the base of k selected criteria. Without a loss of universality it can be assumed that for each criterion holds "the more the better". The goal is to rank the funds in the form of preference structure (P, S, I), or (P, S, I, R) where P means the strict preference, S means a weak preference, I denotes indifference and R denotes incomparability.

There are several classes of methods for solving of such kinds of problems. In the application the method PROMETHE II was used[1]. The method is based on a construction of generalised criteria and indices of multiple criteria preferences. Intensity of one fund preference over the second is a function of the difference in performances according individual criteria and takes a value from 0 to 1. If y and z are two funds from the set Y which are to compare from the view point of criterion i, then

$$d_i = y_i - z_i$$

and the value of preference function

$$F_i\left(y_i, z_i\right) = P\left(d_i\right) = 1 - e^{-\frac{d_i^2}{2\sigma^2}}, \; d_i \geq 0,$$

where σ represents standard deviation, measures the contribution of criterion i to the total preference of y over z. Let us note that this Gaussian preference function is not only possible one. So called *usual criterion*, *quasi-criterion*, *criterion with linear*

[1] Mlynarovič, V.: Modely a metódy viackriteriálneho rozhodovania. Ekonóm, Bratislava 1998

preference level criterion or *criterion with linear preference and indifference area* can also be used.

Let us suppose that for each criterion i a preference function F_i was defined and w_i expresses relative importance of criterion i. Then for all couples of investment funds y a z a following index of multiple criteria preferences is defined

$$\pi(y,z)=\frac{\sum_{i=1}^{k} w_i F_i(y,z)}{\sum_{i=1}^{k} w_i}$$

The index measures investor preference intensity for fund y over fund z in such a way, where all criteria are taken into account simultaneously. From these calculations a matrix of indices can be developed. For each fund y, the mean of preference intensities over all other funds is defined in the form of outgoing flow

$$\Phi^{+}(y)=\frac{\sum_{y\in Y}\pi(y,z)}{n-1},$$

where n is the number of assumed funds. In turn, the mean of preference intensities of all other funds over fund y is defined in the for so called incoming flow

$$\Phi^{-}(y)=\frac{\sum_{z\in Y}\pi(z,y)}{n-1}.$$

Finally the net flow is defined as

$$\Phi(y)=\Phi^{+}(y)-\Phi^{-}(y)$$

and PROMETHEE outranking relationships are defined as:

Fund y outranks fund z iff $\Phi(y) > \Phi(z)$.

Fund y is indifferent to fund z iff $\Phi(y) = \Phi(z)$.

Table 1: *Starting data about funds*

Name of the fund	Currency	Max ZHP-PT	min W-STDEV	max RPS	min MinI	Max V1M	max V3M
SPORO K	SKK	-0.82	0.2782402	1.3	30000	0.28	1.04
IAM KONTO	SKK	0.07	0.1846099	1.3	5000	0.25	0.75
TAM - K	SKK	0.00	0.2044322	1.2	20000	0.20	0.61
TAM - K D	SKK	-0.33	0.1586423	0.7	500000	0.41	1.21
TAM - Euro	SKK	-0.76	0.5929653	0.7	500000	-1.54	0.00
TAM - USD	SKK	-0.58	1.6494903	0.7	500000	1.35	0.00
VÚBAM	SKK	-0.94	0.3601544	0.95	5000	-0.02	0.52
STDEVP		0.3716	0.4929983	0.2644	240146.6	0.795	0.4335

At now the method will be illustrated for selected data on performance a subset of money market funds published on April 4, 2003. For simplicity only a subset of criteria will be also used. The starting data are presented in Table 1, where the following criteria were applied:

ZHP-PT - change of crown value of the share in last week,
W-STDEV - standard deviation of week crown returns for the last 26 weeks,
RPS - charge for fund management for year in %,
MinI - minimum value of the first investment in SKK
V1M - crown return of the fund for last month in %
V3M - crown return of the fund for last 3 months in %.

The final row of the table consist standard deviations of the values of criteria. According to the described methodology for each criterion a preference function must be selected. As it was mentioned above, the Gaussian preference function is selected for all criteria and the standard deviation is only parameter of the function. Results of such decision are presented in Table 2.

Table 2: *Preference functions and parameters values*

	6	6	6	6	6	6
	max	Min	max	min	max	Max
q-indif						
p-pref						
Sigma	0.37156865	0.49299832	0.26438222	240146.638	0.79460398	0.43345156

Data in Table 2 together with data in Table 3 complete the problem formulation of selected investment funds ranking. From the final row of the Table 3 one can see that equal weights were used for all criteria. Remind that the weight can be used for relative importance of individual criteria expressions.

Table 3: *Input data*

	MAX	MIN	MIN	MIN	MAX	MAX
	ZHP-PT	W-STDEV	RPS	MinI	V1M	V3M
SPORO K	-0.82	0.28	1.30	30 000.0	0.28	1.04
IAM KONTO	0.07	0.18	1.30	5 000.0	0.25	0.75
TAM - K	0.00	0.20	1.20	20 000.0	0.20	0.61
TAM - K D	-0.33	0.16	0.70	500 000.0	0.41	1.21
TAM - Euro	-0.76	0.59	0.70	500 000.0	-1.54	0.00
TAM - USD	-0.58	1.65	0.70	500 000.0	1.35	0.00
VÚBAM	-0.94	0.36	0.95	5 000.0	-0.02	0.52
Weights	0.16667	0.16667	0.16667	0.16667	0.16667	0.16667

At now we will illustrate a principle of the method, i.e. how the preference relation between two funds is evaluated when all criteria are taking into account simultaneously. Let us assume the first two funds with names SPORO and IAM KONTO. We will compute the index multiple criteria preference of fund SPORO over fund IAM KNTO. All needed data are in Table 4.

Table 4: *Two funds comparison - computation of the index of multiple criteria preference of* ***SPORO K over IAM KONTO***

Name of the fund	Max ZHP-PT	min W-STDEV	max RPS	min MinI	Max V1M	max V3M
SPORO K	-0.82052	0.27824	1.3	30000	0.278878	1.042836
IAM KONTO	0.067308	0.18461	1.3	5000	0.250458	0.750092
Criterion values difference	-0.88783	-0.09363	0	-25000	0.028421	0.292744
Standard deviations	0.371569	0.492998	0.264382	240146.6	0.794604	0.433452
Gaussian prefer. function	**0**	**0**	**0**	**0**	**0.000639**	**0.20393**
Kriteria weights	0.167	0.167	0.167	0.167	0.167	0.167
Index of multiple kriteria preference of SPORO K over IAM KONTO:					**0.034095**	

Table 5: *Matrix of multiple criteria preference indices and incoming flows*

	SPORO K	IAM KON	TAM - K	TAM-K D	TAM-Eur	TAM-US	VÚBAM
SPORO K	0.00000	0.03409	0.06583	0.14211	0.48484	0.46274	0.10756
IAM KONTO	0.16095	0.00000	0.01179	0.21897	0.63059	0.57007	0.20349
TAM - K	0.16562	0.01151	0.00000	0.19803	0.59098	0.53002	0.17753
TAM - K D	0.27028	0.22923	0.24778	0.00000	0.45664	0.36112	0.33920
TAM - Euro	0.15631	0.15398	0.13879	0.00000	0.00000	0.14990	0.07883
TAM - USD	0.28589	0.25646	0.24725	0.08316	0.18547	0.00000	0.25259
VÚBAM	0.09818	0.09728	0.06041	0.14675	0.39041	0.39411	0.00000
$\Phi(-)$	0.18954	0.13042	0.12864	0.13150	0.45649	0.41132	0.19320

***Table 6:** Outgoing flows and net flows*

	Φ(+)	Φ
SPORO Korunový peňažný fond, o.p.f.	0.216198	0.026659
IAM KONTO - peňažný	0.29931	0.168886
TAM - Korunový peňažný	0.278947	0.150304
TAM - Korunový peňažný **D**ividendový	0.317374	0.185871
TAM - Euro peňažný dividendový fond	0.112968	-0.34352
TAM - Dolárový peňažný dividendový fond	0.218469	-0.19286
VÚBAM - Peňažný korunový fond	0.197857	0.004657

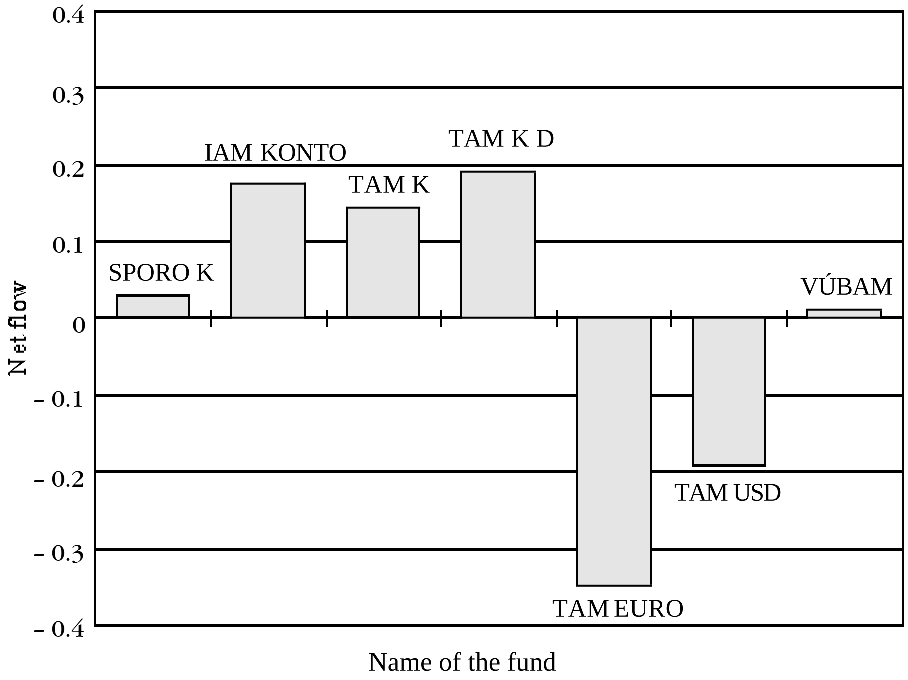
The starting point is differences of criteria values. For example if we compare these two funds according to the first criterion, the corresponding difference is -0.88783. It means that IAM KONTO is better than SPOPRO so the contribution of the criterion to the preference of SPORO over IAM KONTO must equals 0. The same result is valid for the second and the third criterion as well. In this situation however one must take into account that, in the contrary with the first criterion, for these two criterion we have "the less the better". At the fourth criterion the difference is positive (0.028421). It means that this criterion contributes with a positive value to the total preference of SPORO over IAM KONTO. The contribution is computed according to the selected preference function, i.e.

$$1-e^{-\frac{d_i^2}{2\sigma^2}} = 1-e^{-\frac{0.028421^2}{2\times 0.794604^2}} = 0.000639.$$

For the last criterion the contribution is 0.203939 and the resulted value of index is the average sum of these values, namely

$$0.167\times 0.000639 + 0.167\times 0.20393 = 0.034095.$$

All couples of the funds must be compared in this way. Results provide the matrix o the multiple criteria indices that is shown in Table 5. At the first row and the second column of the matrix one can find the computed value 0.03409. The row with the name Φ(-) consists in coming flows. The higher incoming flow, the worse for the fund. The column Φ(+) in Table 6 consists outgoing flows. The higher outgoing flow, the better for the fund. The resulted ranking of the funds is indicated with the column **Φ** in Table 6 that resents the net flows. The higher net flow the better for the fund. It is worth to see that the sum of the net flows always equals zero. It gives not only the chance to outrank funds, but also to divide them into two groups. Relatively good are funds with positive values of the net flows and relatively bad - funds with negative values of net flows. Funds outranking is illustrated in Picture 1.



Picture 1: Selected money market funds outranking

2. RESULTS

An application of described methodology for outranking of money market funds, bonds market funds and equities market funds on the base of weekly data provides three types of results[2] :

- funds outranking on the base of the current week data,
- average results for the last ten weeks,
- results that presents long-term tendencies of funds performance developments.

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2 The *Sanna* software package developerd at University of Economics in Prague by *J. Jablonsky* was used for computations

***Table 7:** Money market funds outranking in current week*

Fund	Currency	Net flow
SPORO Korunový peňažný fond, o.p.f.	SKK	0.274928
VÚBAM - Peňažný korunový fond	SKK	0.198316
IAM KONTO - peňažný	SKK	0.193162
KBC - Multi Cash CSOB SKK	SKK	0.152276
TAM - Korunový peňažný dividendový	SKK	0.103357
TAM - Korunový peňažný	SKK	0.046587
KBC - Multi Cash EURO	EUR	0.02583
KBC - Multi Cash EURO Medium	EUR	-0.01922
CI - Euro Cash (výnosový)	EUR	-0.03801
CI - Euro Cash (plne rastový)	EUR	-0.03906
TAM - Europeňažný dividendový fond	SKK	-0.05902
KBC - Multi Cash CSOB CZK	CZK	-0.07749
KBC - Multi Cash CAD	CAD	-0.08831
CI - Euro Cash (rastový)	EUR	-0.09505
KBC - Multi Cash CAD Medium	CAD	-0.11365
TAM - Dolárový peňažný dividendový fond	SKK	-0.11798
KBC - Multi Cash USD	USD	-0.16835
Živnobanka - Sporokonto	CZK	-0.17833

As an illustration results for money market funds will be presented. The current weekly results on May 5, 2003 are presented in Table 7. Corresponding funds outranking n the base the last ten weeks is presented in Table 8. The table contents together with values of average net flows for the last ten weeks also standard deviations of these values for the same period. These values measure volatilities of results for the period and provide a measure of risk. The combination of these two results leads to a construction of so called efficient funds boundary that consists of funds where a better average result can be achieved only with a higher risk. Such constructions create starting points for modern portfolio theory applications in decisions concern assumed investment opportunities space.

Table 8: *Funds outranking on the base of the last ten week average*

Fund	Currency	Average net flow	Standard deviation
VÚBAM - Penažný korunový fond	SKK	0.3046065	0.0720216
SPORO Korunový penažný fond, o.p.f.	SKK	0.2987799	0.0282207
TAM - Korunový penažný	SKK	0.2062471	0.0716619
IAM KONTO - penažný	SKK	0.1589754	0.038912
KBC - Multi Cash CSOB SKK	SKK	0.1059544	0.0737879
TAM - Korunový penažný dividendový	SKK	0.004899	0.0472015
KBC - Multi Cash EURO	EUR	-0.0242455	0.0332315
KBC - Multi Cash CAD	CAD	-0.039446	0.066714
KBC - Multi Cash CAD Medium	CAD	-0.0428864	0.0679084
CI - Euro Cash (výnosový)	EUR	-0.0537313	0.0219604
CI - Euro Cash (plnerastový)	EUR	-0.06101	0.0035195
KBC - Multi Cash EURO Medium	EUR	-0.0633731	0.0359298
TAM - Europenažný dividendový fond	SKK	-0.0805186	0.0308379
KBC - Multi Cash CSOB CZK	CZK	-0.0874972	0.040199
TAM - Dolárový penažný dividendový fond	SKK	-0.1106798	0.0415547
KBC - Multi Cash USD	USD	-0.124524	0.0373659
CI - Euro Cash (rastový)	EUR	-0.1578926	0.0362218
Živnobanka-Sporokonto	CZK	-0.233658	0.0165638

Funds performance from a long term view can be described with sequences of average ten week results for the whole period of evaluation that starts on January 1, 2002. Such kind of results is illustrated in Picture 2.

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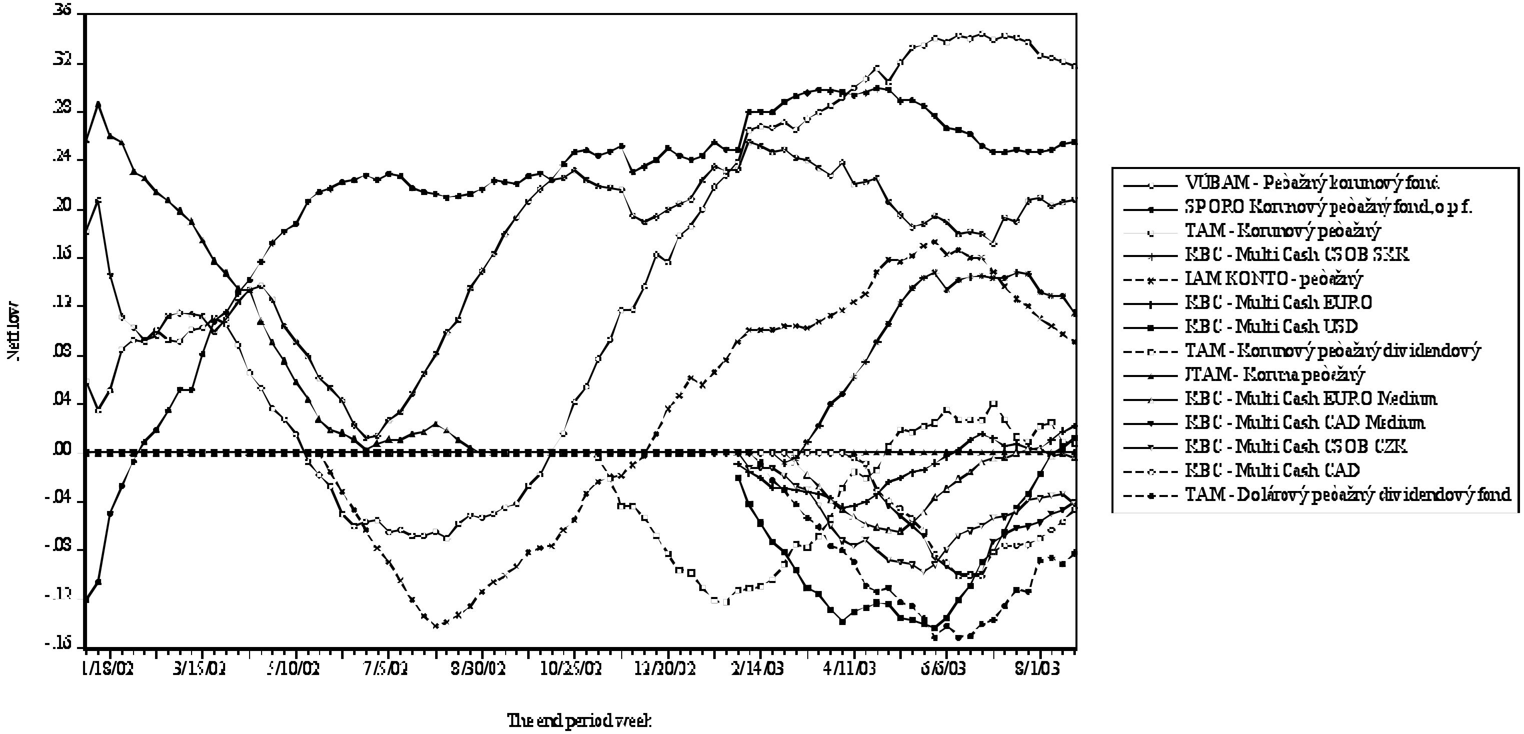

Picture 2: Long - term money market funds performance

MULTI CRITERIA OPTIMIZATION IN TRANSPORT SCHEDULING[1]

STANISLAV PALÚCH

Abstract The fundamental vehicle-scheduling problem (VSP) is obviously formulated as a matching problem for which there exists a polynomial algorithm. However, real life scheduling problems have to comply with many additional requirements which can be formulated as nonlinear constraints. This paper shows how these constraints can be converted into objectives. Thus the general vehicle scheduling problem can be formulated as a matching problem with nonlinear objective function. For such problem there exists a good heuristic algorithm.

0. INTRODUCTION

Transport scheduling problems are frequently discussed in world scientific literature. Vehicle and crew scheduling in regional bus transport in Czech and Slovak Republic is different from the one in many other countries so the corresponding mathematical modelling has many specific properties (see Cerná in [2]).

1. VEHICLE SCHEDULING

Trip s is a travel from a starting point to a finishing point of a route and is considered to be an elementary amount of the work of a bus. *Trip s* can be considered as an arbitrary quadruple $s = (dp, ap, dt, at)$ where dp, ap are the departure place and arrival place of the trip s and dt; at are departure time and arrival time of the trip s. We will say that the trip s_i *precedes* the trip s_j and we will write $s_i \prec s_j$ if the trip s_j can be linked after the trip s_i into a running board for one bus. Relation $\prec$ is irreflexive and transitive.

Running board, or *running board of a bus* is an arbitrary nonempty sequence $T = s_1, s_2, \ldots, s_m$ of trips with property:

$$s_1 \prec s_2 \prec s_m. \tag{1}$$

The number m we will call *length of running board T*. We will write $s_i \rightarrow s_j$ if both trips s_i, s_j are provided by the same bus and the trip s_j is linked immediately behind the trip s_i. Note that $s_i \rightarrow s_j$ implies $s_i \prec s_j$. For any given set $S = \{s_1, s_2, \ldots s_n\}$ of

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1 This paper was supported by VEGA as a grant No. 1/0490/03.

trips with precedence relation $\prec$ we can construct a bus schedule. *Bus schedule* of the set S of trips is a set of running boards $O = \{T_1, T_2, \dots T_k\}$ of the form

$$\begin{aligned} T_1 &= s_{1,1} \to s_{1,2} \to \dots \to s_{1,n(1)-1} \to s_{1,n(1)} \\ T_2 &= s_{2,1} \to s_{2,2} \to \dots \to s_{2,n(2)-1} \to s_{2,n(2)}, \\ &\vdots \\ T_k &= s_{k,1} \to s_{k,2} \to \dots \to s_{k,n(k)-1} \to s_{k,n(k)} \end{aligned} \tag{2}$$

such that every trip of the set S occurs exactly in one running board of O. To every bus schedule $O = \{T_1, T_2, \dots T_k\}$ the number $C(O)$ - the cost of bus schedule O is assigned. We will say that the cost $C(O)$ is separable if it can be expressed as a sum of costs of all running boards T_1, T_2, ..., T_k, i. e.

$$C(O) = \sum_{i=1}^{k} c(T_i), \tag{3}$$

where $c(T_i)$ denotes the cost of running board T_i. In most simple cases the cost of running board $T = s_1 \to s_2 \to \dots \to s_{m-1} \to s_m$ is the sum of all linkage costs:

$$c(T) = \sum_{i=1}^{m-1} c(s_i, s_{i+1}). \tag{4}$$

The linkage cost denoted $c(s_i, s_j)$ represents namely deadmileage expenses, however, it may include waiting costs, line changing penalty and other penalties as well. In this case we will say that the cost $c(T)$ is linear. We will say that the cost $C(O)$ of bus schedule O is linear, if C is regular and the cost c of running board is linear.

Most important objectives for vehicle scheduling are the following:

1. **O1:** Minimization of the number of running boards.
2. **O2:** Minimization of the total linkage cost.

Fundamental vehicle scheduling problem FVSP is to find a bus schedule with the minimum total linear cost from all bus schedules with the minimum number of running boards. In the terms of bivalent programming the FVSP can be formulated as follows:

Let $d_{ij} = c(s_i, s_j)$ if $s_i \prec s_j$ and $d_{ij} = \infty$ otherwise. Let x_{ij} be a zero-one decision variable saying that if $x_{ij} = 1$ and $d_{ij} < \infty$, then s_i immediately follows s_j in some running board T. To solve FVSP means

Minimize $$\sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \tag{5}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \text{for } i = 1,2,\ldots,n \tag{6}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \text{for } j = 1,2,\ldots,n \tag{7}$$

$$x_{ij} \in \{0,1\} \tag{8}$$

We see that we got a classical assignment problem which can be solved in polynomial time. (Several related problem of fleet size minimization see in Cerná [1], Cerný [3] and Peško [5].)

Remark: We can change the definition of d_{ij} as follows: $d_{ij} = c(s_i, s_j)$ if $s_i \prec s_j$ and d_{ij} = the fixed costs of one bus per day. Then the optimization will give us also an optimum number k of vehicles.

2. SIMULTANEOUS VEHICLE AND CREW SCHEDULING

The approach to the vehicle and crew scheduling in many countries is a two step procedure: first to calculate an optimum bus schedule and then to find an optimum crew schedule for the bus schedule calculated in first step. In the resulting schedule any driver can drive any bus.

In Czech and Slovak Republic there are closer ties between drivers and vehicles. One bus schedule has to be covered by one or two crew schedules. In this case the considered bus running board has to fulfill several conditions. The most important one of them is so called safety break constraint (SB) which is defined by law. We will say that running board fulfills safety break condition (SB) or is feasible if in every time interval 240 minutes long there exists at least 30 minutes of safety break. This safety break can be in one continuous piece or two or three time intervals, every one of them is at least 10 minutes long.

The most important constraints for vehicle and crew scheduling are

1. **C1:** All vehicles have to return after work to the places where they started in the morning. (As the starting points can be given one depot, several depots or simply a requirements that every bus returs after finishing the daily work to the starting point of corresponding running board.)
2. **C2:** All running boards have to fulfill the safety break condition.
3. **C3:** The time length of every running board must be suitable for one crew or two crews. This means that it must be in time interval if it is designed for one driver or in interval for two drivers.

Just mentioned constraints can not be expressed as the assignment problem with several additional linear constraints. The idea of the proposed model is to formulate such conditions in the objective function.

Assume that $T = s_1 \to s_2 \to \ldots \to s_{m-1} \to s_m$. The constraint **C1** can be modelled by objective function

$$c_1(T) = c(s_m, s_1), \tag{9}$$

where $c(s_m, s_1)$ represents all expenses connected with the move of the vehicle from arrival place of the trip s_m to departure place of the trip s_1.

The modelling of safety break conditrion **C2** can be achieved by the following objective function $c_2(T)$:

$$c_2(T) = \begin{cases} 0 & \text{if } T \text{ contains a feasible safety break} \\ \infty & \text{otherwise} \end{cases} \tag{10}$$

Assign $t = at_m - dt_1$, where at_m is the arrival time of the last trip of T and dt_1 is the departure time of the first trip of T. The value t expresses the time length of the running board T. The working time constraints **C3** are expressed by objective $c_3(T)$:

$$c_3(T) = \begin{cases} 0 & \text{if } t \in \left(t_1^1, t_2^1\right) \text{ or } t \in \left(t_1^2, t_2^2\right) \\ \infty & \text{otherwise} \end{cases} \tag{11}$$

Simultaneous vehicle and crew scheduling problem SVCSP can be formulated as a multicriteria assignment problem with objectives $\sum_i c(T_i)$, $\sum_i c_1(T_i)$, $\sum_i c_2(T_i)$, $\sum_i c_3(T_i)$. All mentioned objectives are separable, all except the first one are nonlinear and their formulation by means of mathematical programming is rather complicated. We can define the complex objective function

$$C(O) = \sum_i c(T_i) + K_1 \sum_i c_1(T_i) + K_2 \sum_i c_2(T_i) + K_3 \sum_i c_3(T_i) \tag{12}$$

and to formulate SVCSP as to find a bus schedule with minimum value of $C(O)$. Arising mathematical problem is probably NP-hard and therefore suboptimal methods are to be used for solving SVCSP.

There are many other requirements from practice which can be modelled in similar way. Constraints similar to **C1** studied e.g. Peško in [4].

3. HEURISTICS FOR SVCSP

For every bus schedule O we can define the set $N(O)$ of neighbouring bus schedules. There are several possibilities how to define $N(O)$ by combining running boards from O. The simplest one is to combine the trips of a pair of running boards, more sophisticated methods combine heads and tails of all running boards.

The following procedure was proposed for finding suboptimal solution of SVCSP. First find an (exact) optimum bus schedule $O = \{T_1, T_2, \ldots T_k\}$ with objective $\sum_i c(T_i)$. Hungarian method or max-flow-min-cost algorithm can be used for solving corresponding assignment problem. The bus schedule O will be the starting solution for the following neighbourhood search procedure.

Step 1: If for all $\overline{O} \in N(O)$ $C(\overline{O}) \geq C(O)$ STOP, O is a suboptimal bus schedule.

Step 2: Find $\overline{O} \in N(O)$ such that $C(\overline{O}) < C(O)$.

Step 3: Set $O := \overline{O}$, actualize $N(O)$ and GOTO Step 1.

The objective functions (10), (11) appeared not to be suitable for just described algorithm, since they do not express "how far" is actual solution O from the ideal one. In all practical cases the following objectives showed to be much more usefull

$$\bar{c}_2(T) = \min\{\Delta \geq 0;\ \text{such that every time interval } (t, t + 240 + \Delta) \text{ in } T \text{ contains at least 30 min. safety break.}\} \tag{13}$$

$$\bar{c}_3(T) = \begin{cases} 0 \ \ if \ \ (t_1^1, t_2^1) \ \ or \ \ (t_1^2, t_2^2) \\ t_1^1 - t \quad if \quad t < t_1^1 \\ \min\{(t - t_2^1), (t_1^2 - t)\} \quad if \quad t \in (t_2^1, t_1^2) \\ t - t_2^2 \quad if \quad t > t_2^2 \end{cases} \tag{14}$$

This approach was used for optimization of regional bus transport in many real world cases with great economical gain. The dead mileage savings were from 6 to 20 %, the number of vehicles dropped by up to 15 %.

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USING SEMIDEFINITE PROGRAMMING FOR THE CHANNEL ASSIGNMENT PROBLEM

ŠTEFAN PEŠKO

Abstract We investigate the problem of assigning channels (codes) to the cell of cellular mobile network so that we avoid interference and minimize the number of channels used. The channel assignment problem is inspired by this problem. We propose a heuristic based on the exact algorithms for the standard semidefinite and linear programming.

1. INTRODUCTION

The demands for mobile telephone communication are connected with the limited range of the frequency spectrum. Optimal frequency assignment is an increasing problem in order to use the available frequency spectrum with optimum efficiency.

The problem was first solved by a graph colouring algorithm by Hale [Hal]. The generalisation of the graph colouring, inspired by this problem, is known as the *channel assignment problem* (CAP). Various approximate algorithms (see [Ba1], [Ba2]) and exact algorithms by Kral [Kra] have been proposed. The graph colouring problem i.e. determining the *chromatic number* of a graph is NP-hard (see [Ben]) and so the channel assignment problem is also NP-hard and therefore an optimal assignment cannot be found in reasonable polynomial time.

We formulate the relaxed CAP in terms of a *semidefinite programming* (SDP) and we obtain a solution of the SDP. Then we map this solution of the SAP back into a valid solution for CAP. This algorithm can lose precision in this process. We propose a heuristic based on the exact algorithms for the standard semidefinite and linear programming.

2. SEMIDEFINITE PROGRAMMING

We will apply semidefinite programming to our problem. We need some definitions and facts [Wol] to understand this entails.

Definition 1. A matrix $\mathbf{A} \in \Re^{n\times n}$ is a *positive semidefinite* if

(1) $\quad \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} \geq 0 \quad \forall \mathbf{x} \in \Re^{n\times n}$.

Fact 1. For a symmetric matrix $\mathbf{A} \in \Re^{n\times n}$ the following are equivalent:

1. $\mathbf{A}$ is a positive semidefinite.
2. $\mathbf{A}$ has only the nonnegative eigenvalues.

3. $\mathbf{A} = \mathbf{B}^T.\mathbf{B}$ for some matrix $\mathbf{B} \in \Re^{n\times n}$.

We will note $\mathbf{A} \bullet \mathbf{B} = \sum_i \sum_j a_{ij} b_{ij}$ for the *inner product*, $\mathbf{A.B} = \sum_j a_{ij} b_{jk}$ for the *matrix product* of two matrices $\mathbf{A}, \mathbf{B} \in \Re^{n\times n}$ and $\mathbf{a.b} = \sum_i a_i b_i$ for the *scalar product* of two vectors $\mathbf{a}, \mathbf{b} \in \Re^n$.

Now we can define following generalisation of the linear programming.

Definition 2. The (standard) problem of the *semidefinite programming* (SDP) is formulated for given symmetric matrices $\mathbf{C}, \mathbf{A_k} \in \Re^{n\times n}$, a vector $\mathbf{b} \in \Re^n$ and a variable symmetric matrix $\mathbf{X} \in \Re^{n\times n}$ in the form

(2) $\mathbf{C} \bullet \mathbf{X} \to MINIMUM.$

(3) $s.t.\ \mathbf{A_k} \bullet \mathbf{X} = b_k \quad k = 1,2,\dots,m$

(4) X *is positive definite*

Using 3[th] item of the fact 1 we know that there is a matrix $\mathbf{Z} \in \Re^{n\times n}$ such that $\mathbf{X} = \mathbf{Z}^T.\mathbf{Z}$. Let $\mathbf{Z} = (\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n)$ where $\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n$ are columns of $\mathbf{Z}$. Then the problem of the SDP can be rewritten as:

(5) $$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \left(z_i \cdot z_j \right) \to MINIMUM.$$

(6) $$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ijk} \left(z_i \cdot z_j \right) = b_k \quad k = 1,2,\dots,m$$

(7) $$z_i \in \Re^n \quad i = 1,2,\dots,m$$

Fact 2. SDP is polynomial time solvable to within an additive error of ε.

3. CHANNEL ASSIGNMENT PROBLEM

In the context of graph model follows formal definition of the channel assignment problem.

Definition 3. Natural weighted (simple) graph $G = (V, E, d)$ where $V = \{1,2,\dots, n\}$ is a vertex set, $E \subseteq V \times V$ is edge set and positive integer weight of edges $d: E \to \{1,2,\dots\}$ is given.

A *channel assignment* for natural weighted graph G is a function $f: V \to \{0,1,2,\dots\}$ (assignment), the vertexes to nonnegative integers, satisfying the condition

(8) $$|f(\mathrm{u}) - f(\mathrm{v})| \geq d(\mathrm{uv}) \quad \forall \mathrm{uv} \in E.$$

The *span* S(f) of a channel assignment f of the natural weighted graph G is the highest channel assigned by f

(9) $$\mathrm{S}(f) = \max\{ f(v) : v \in V \}.$$

The *Channel Assignment Problem* (CAP) consists of finding a channel assignment f^* with minimised span i.e.

$$(10) \qquad S\left(f^*\right) = \min_f S(f).$$

In our definition of the channel assignment we restrict to the assignment one number (represented channel) to the vertex only. In real-word problems [Ba1],[Ba2] the assignment to the vertex is a set of nonnegative integers (channels).

Note that if we set $d(uv) = 1$ for $uv \in E$ then the CAP is classical colouring problem and $S(f^*)$ is the chromatic number of the graph G.

4. RELAXED CHANNEL ASSIGNMENT PROBLEM

Now we will formulate the relaxed CAP in terms of the SDP and obtain a solution of the SDP which is within $1+\varepsilon$ the optimal. We map the vertexes $v_i \in V$ of the graph G to unit vectors $\mathbf{z}_i \in \Re^n$ and solve the following RCAP:

$$(11) \qquad \alpha \rightarrow MINIMUM$$

$$(12) \qquad z_i \cdot z_i = 1 \quad \forall v_i \in V$$

$$(13) \qquad z_i \cdot z_j = 1 \quad \forall v_i v_j \in E$$

$$(14) \qquad z_i \cdot z_j \geq -\frac{1}{d\left(v_i v_j\right)^2} \quad \forall v_i v_j \in E$$

$$(15) \qquad z_i \in \Re^n \quad \forall v_i \in V$$

The constraint (12) assigns the unit vector to the every vertex of the natural graph. The variable in the constraint (13) is an upper negative bound of the differences of the linked channels.

From the negative part of the condition (8) after the substitution

$$(16) \qquad f\left(v_i\right) - f\left(v_j\right) = 1/\left(\mathbf{z}_i . \mathbf{z}_j\right) \quad \forall v_i v_j \in E$$

we have constrain (14). The positive part of the condition (8) is implicitly satisfied via constraint (13) -- it is possible show, that $\alpha < 0$. The goal function (11) minimizes the highest difference of the linked channels and not the span. So the solution of RCAP approximates of the optimal channel assignment only.

Note that $\mathbf{z}_i \cdot \mathbf{z}_j = \left|\mathbf{z}_i\right|\left|\mathbf{z}_j\right|\cos\delta_{ij}\left| \leq 1\right.$ and for the colour problem (with $d(v_i v_j) = 1$ for $v_i v_j \in E$) is the constrain (14) implicitly satisfied and this constrain can be omitted. This formulation is known as the *vector colouring problem* and studied in [Kar],[Ben].

5. APPROXIMATE SOLUTION

We do not actually find the solution -- the channel assignment, when we solve RCAP. From the solution ($\mathbf{z}_1$, $\mathbf{z}_2$, ..., $\mathbf{z}_n$) we must determine the assignment (f(v_1), f(v_2),..., f(v_n)) satisfying the condition (8).

Let *induced digraphs* $G_k = (V,E_k)$ for k =1, 2, ..., *n* have the oriented sets of the edges of graph *G* according to the vectors ($\mathbf{z}_1$, $\mathbf{z}_2$, ..., $\mathbf{z}_n$) in the form

$$E_k = \left\{ \left(v_i, v_j \right) : \mathbf{z}_k \cdot \mathbf{z}_i \geq \mathbf{z}_k \cdot \mathbf{z}_j, v_i v_j \in E \right\} \cup \left\{ \left(v_i, v_j \right) : \mathbf{z}_k \cdot \mathbf{z}_i < \mathbf{z}_k \cdot \mathbf{z}_j, v_i v_j \in E \right\}.$$

For every digraph G_k we can solve in the polynomial time the following linear program LP:

(17) $\beta_k \rightarrow MINIMUM$

(18) $s.t.\; y_i - y_j \geq d\left(v_i v_j\right) \quad \forall \left(v_i, v_j\right) \in E$

(19) $0 \leq y_i \leq \beta_k \quad \forall v_i \in V$

It is possible show that integer solution $\left(\lfloor y_1 \rfloor, \lfloor y_2 \rfloor, \dots, \lfloor y_n \rfloor\right)$ is feasible solution of the LP with the unabated goal value β_k. Than we can define $f\left(v_i\right) = \lfloor y_1 * \rfloor$, $v_i \in V$ where $\left(\lfloor y_1 * \rfloor, \dots, \lfloor y_n * \rfloor\right)$ is the best LP solution with heuristic span S(f) = β^* = min{β_k: k = 1, 2, ..., n}.

6. EXAMPLE

In figure 1 we have instance for the channel assignment problem modelled by graph *G*.

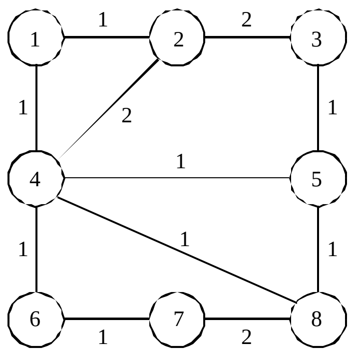

*Fig.1 Natural weighted graph G=(V,E,*d*)*

We use SeDuMi [Sed] - an toolbox for MATLAB, witch solve optimisation problems with linear, quadratic and semidefiniteness constraints. The corresponding RCAP has optimal solution $\mathbf{X} = (x_{ij})$ where $x_{ij} = \mathbf{z}_i \cdot \mathbf{z}_j$

$$\mathbf{X}=\begin{pmatrix} 1.0000 & -0.2500 & 0.0547 & -0.2500 & 0.0393 & 0.0841 & -0.0560 & 0.0946 \\ -0.2500 & 1.0000 & -0.2500 & -0.2500 & 0.1418 & 0.0812 & -0.0392 & 0.0434 \\ 0.0547 & -0.2500 & 1.0000 & 0.1420 & -0.2500 & -0.0418 & -0.0002 & 0.0535 \\ -0.2500 & -0.2500 & 0.1420 & 1.0000 & -0.2500 & -0.2500 & 0.1489 & -0.2500 \\ 0.0393 & 0.1418 & -0.2500 & -0.2500 & 1.0000 & 0.0527 & 0.0482 & -0.2500 \\ 0.0841 & 0.0812 & -0.0418 & -0.2500 & 0.0527 & 1.0000 & -0.2500 & 0.1440 \\ -0.0560 & -0.0392 & -0.0002 & 0.1489 & 0.0482 & -0.2500 & 1.0000 & -0.2500 \\ 0.0946 & 0.0434 & 0.0535 & -0.2500 & -0.2500 & 0.1440 & -0.2500 & 1.0000 \end{pmatrix}$$

In figure 2 we have induced digraph G_4 generated by 4th column of the matrix X.

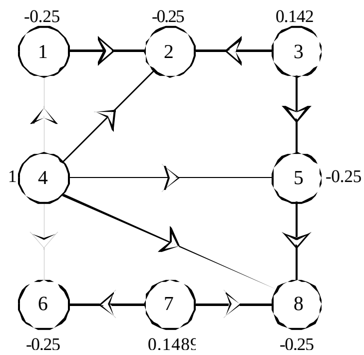

Fig. 2 Induced digraph G4

The corresponding LP has optimal solution **y** =(1.0, 0.0, 2.0, 2.0, 1.0, 0.4377, 2.0, 0.0) with channel assignment (f(1), f(2), ..., f(8)) = (1,0,2,2,1,0,2,0) in figure 3.

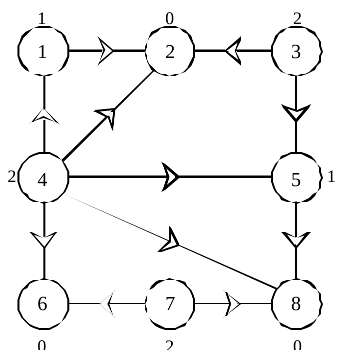

Fig. 3 Channel assignment in G4

This is the best solution with span S(*f*) = 2.

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MODEL OF LIFE INSURANCE POLICIES USING MARKOV CHAINS WITH REWARDS

MILAN SITAŘ

Abstract In this article classical approach for calculating premium and reserve of life insurance products is replaced by an alternative method using Markov chains with rewards. This approach allows us to investigate more complicated problems. Recursive formulas for mathematical reserves of life insurance products considered as expected total discounted return of a Markov reward chain are obtained. Using specific properties of the reserves we can also calculate the insurance premium. Moreover, recursive formulas for the variance of the reserve are obtained.

1. INTRODUCTION

In this note, we consider a discrete time Markov reward process with finite state space. We assume that the rewards associated with the transitions are random variables. We are interested in properties of cumulative reward earned in the subsequent transitions of the Markov chain. Similarly as in [1], and [3] for the finite horizon models recursive formulas for expected value and variance of the cumulative (random) discounted reward are investigated (section 2). These formulas are used to calculate some life insurance products (section 3). Comparing with classical results (cf. e.g. [2]), this approach allows to investigate more complicated models. A simple example illustrates the methods and results (section 4).

2. MARKOV CHAINS WITH REWARDS

Let $X = \{X_n, n = 0,1,\dots\}$ be a nonhomogeneous Markov chain with finite state space $\mathbf{S}=\{1,2,\dots S\}$, initial distribution $\boldsymbol{p}(0)$ and the transition probability matrix $\boldsymbol{P}(n) = [p_{ij}(n)]_{i,j=1}^{S}$ at time n (obviously, $\sum_{j=1}^{S} p_{ij}(n) = 1$, $p_{ij}(n) \geq 0$).

We shall assume that if state $j \in \mathbf{S}$ is reached at time $n+1$ from state $i \in \mathbf{S}$ an immediate (random) reward $\xi_{ij}(n)$ is earned which is finite almost surely. $r_{ij}(n)$, resp. $s_{ij}(n)$, denotes the first, resp. the second, moment of the immediate reward. Obviously, the expected reward in state i at time n is equal to $r_i(n) = \sum_{j=1}^{S} p_{ij}(n) r_{ij}(n)$ and the second moment of the reward earned in state i is equal to $s_i(n) = \sum_{j=1}^{S} p_{ij}(n) s_{ij}(n)$. The symbol $\boldsymbol{r}(n)$, resp. $\boldsymbol{s}(n)$ denotes the $S \times 1$ vector whose i-th element equals $r_i(n)$, resp. $s_i(n)$, and $\boldsymbol{R}(n)=[r_{ij}(n)]$, resp. $\boldsymbol{S}(n)=[s_{ij}(n)]$, is an $S\times S$ matrix.

If we assume that $\eta\in(0,1)$ is the discount factor (for the corresponding interest rate ρ we have $\rho=\frac{1-\eta}{\eta}\Leftrightarrow\eta=\frac{1}{1+\rho}$) then we denote by $\boldsymbol{v}(\eta,m,n)$, resp. by $\boldsymbol{u}(\eta,m,n)$, the $S\times1$ vector of the first, resp. of the second, moments of the discounted rewards earned after the m-th transition in the $(n-m+1)$-next transitions and discounted to time m (its i-th element, denoted $v_i(\eta,m,n)$, resp. $u_i(\eta,m,n)$, is the first, resp. the second, moment of the total discounted reward earned provided the chain starts in state $i\in\mathbf{S}$). Obviously

$$v_i(\eta,m,n)=\mathrm{E}\left[\sum_{k=m}^{n}\eta^{k-m}\xi_{X_k,X_{k+1}}\mid X_m=i\right],$$

resp.

$$u_i(\eta,m,n)=\mathrm{E}\left[\left(\sum_{k=m}^{n}\eta^{k-m}\xi_{X_k,X_{k+1}}\right)^2\mid X_m=i\right],$$

and for the corresponding variance it holds

$$[\sigma_i(\eta,m,n)]^2=u_i(\eta,m,n)-[v_i(\eta,m,n)]^2.$$

In what follows we derive recursive relations for $\boldsymbol{v}(\eta,m,n)$ and $\boldsymbol{u}(\eta,m,n)$:

$$\begin{aligned}\boldsymbol{v}(\eta,m,n)&=\mathrm{E}\left[\sum_{k=m}^{n}\eta^{k-m}\xi_{X_k,X_{k+1}}\mid X_m\right]=\\&=\mathrm{E}\left[\xi_{X_m,X_{m+1}}\mid X_m\right]+\eta\boldsymbol{P}(m)\mathrm{E}\left[\sum_{k=m+1}^{n}\eta^{k-m-1}\xi_{X_k,X_{k+1}}\mid X_{m+1}\right]=\\&=\boldsymbol{r}(m)+\eta\boldsymbol{P}(m)\boldsymbol{v}(\eta,m+1,n)\end{aligned}\tag{1}$$

$$\begin{aligned}\boldsymbol{u}(\eta,m,n)&=\mathrm{E}\left[\left(\sum_{k=m}^{n}\eta^{k-m}\xi_{X_k,X_{k+1}}\right)^2\mid X_m\right]=\\&=\mathrm{E}\left[\left(\xi_{X_m,X_{m+1}}\right)^2\mid X_m\right]+2\eta\mathrm{E}\left[\xi_{X_m,X_{m+1}}\left(\sum_{k=m+1}^{n}\eta^{k-m-1}\xi_{X_k,X_{k+1}}\right)X_m\right]+\\&+\eta^2\boldsymbol{P}(m)\mathrm{E}\left[\left(\sum_{k=m+1}^{n}\eta^{k-m-1}\xi_{X_k,X_{k+1}}\right)^2\mid X_{m+1}\right]=\end{aligned}$$

$$=\boldsymbol{s}(m)+2\eta\Big[\boldsymbol{P}(m)[\boldsymbol{v}(\eta,m+1,n)]_{diag}\mathbf{R}^{\mathrm{T}}(m)\Big]_{diag}\boldsymbol{e}+\eta^2\boldsymbol{P}(m)\boldsymbol{u}(\eta,m+1,n),\tag{2}$$

where $\boldsymbol{e}$ is a unit column vector, $[\boldsymbol{A}]_{diag}$ results from matrix $\mathbf{A}$ by setting off-diagonal entries equal to 0, for a vector $\boldsymbol{b}$, $[\boldsymbol{b}]_{diag}$ is reserved for the corresponding diagonal matrix and $\boldsymbol{A}^{\mathrm{T}}$ denotes the transpose of $\mathbf{A}$. If $\boldsymbol{b}_{sq}$ results from $\boldsymbol{b}$ by squaring each entry, for the variance of the total discounted reward we have

$$[\sigma(\eta, m, n)]_{sq} = \boldsymbol{s}(m) + 2\eta\Big[\boldsymbol{P}(m)[\boldsymbol{v}(\eta, m+1, n)]_{diag}\,\mathbf{R}^{\mathrm{T}}(m)\Big]_{diag}\boldsymbol{e} + \\ + \eta^2\boldsymbol{P}(m)[\boldsymbol{v}(\eta, m+1, n)]_{sq} - \boldsymbol{v}(\eta, m, n)_{sq} + \eta^2\boldsymbol{P}(m)[\sigma(\eta, m+1, n)]_{sq} \tag{3}$$

3. CALCULATION OF LIFE INSURANCE

We focus our attention on life insurance products which creates collective reserve, e.g. endowment, pure endowment or term, and thus they are not flexible and variable as the Universal Life or Unit Linked products. For this moment we do not distinguish if the premium is paid singly or regularly.

To this end, we shall consider Markov chain with three states in which the policy can occur. We assume that the policyholder is alive when the policy is accepted (thus the first state is L - as living). Almost all policies insure the policyholder for the case of death (the second state is D). When the insurance event comes or the policy claim ends then the policy is set aside of the portfolio of insurance policies (the third state is A). Thus the state space

$$\mathbf{S} = (L, D, A)$$

The insurance company calculate some expenses corresponding to the state which Markov chain visits. When the chain is "living" then the policyholder must pay purchase cost and initial commission to the distributor of policy on the first day of the policy term (initial expenses α) or administrative expenses later (renewal expenses β). When the premium is paid, encashment fee must be covered (expenses γ). On the contrary, when the insurance company pay to the policyholder (e.g. the life annuity), the expenses δ are calculated. When Markov chain visits state "dead" and the sum assured is paid to the policyholder, δ expenses are supposed. If the policy is set aside of the portfolio the company has no costs with the policy.

Every life insurance company uses mortality tables for estimating probability of death of a policyholder. If we suppose that our policyholder's entry age is x, we denote such probability q_x (put $p_x = 1 - q_x$). Then we can produce the transition probability matrix along with the transition reward vector of the considered Markov chain with rewards. Obviously at time $n \geq 0$,

$$\boldsymbol{P}(n) = \begin{pmatrix} p_{x+n} & q_{x+n} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$\boldsymbol{R}(n)=\begin{pmatrix}\left(P_n-\alpha_n-\beta_n-\gamma_n-K_n^L-\delta_n^L\right)\boldsymbol{e}^{\mathrm{T}}\\ \left(-K_n^D-\delta_n^D\right)\boldsymbol{e}^{\mathrm{T}}\\ 0\boldsymbol{e}^{\mathrm{T}}\end{pmatrix} \tag{5}$$

where P_n is the premium, K_n^L, resp. σ_n^L, is the sum assured, resp. expenses, when the policyholder is living at time n and the sum assured is paid, K_n^D, resp. σ_n^D, is the sum assured, resp. expenses, in the case that the policyholder dies in period n and the sum assured is paid (similarly $\alpha_n, \beta_n, \gamma_n$ are expenses at time n). All rewards are deterministic. Obviously,

$$\boldsymbol{r}(n)=\begin{pmatrix}P_n-\alpha_n-\beta_n-\gamma_n-K_n^L-\delta_n^L\\ -K_n^D-\delta_n^D\\ 0\end{pmatrix} \tag{6}$$

At time $n=0$ we suppose that the policyholder is alive, hence

$$\boldsymbol{p}(0)=\begin{pmatrix}1\\0\\0\end{pmatrix} \tag{7}$$

If we denote a policy term by N then according to (1) we obtain the first moment of the total discounted reward at time m:

$$\boldsymbol{v}(\eta,m,N)=\boldsymbol{r}(m)+\eta\boldsymbol{P}(m)\boldsymbol{v}(\eta,m+1,N)=$$
$$=\begin{pmatrix}P_m-\alpha_m-\beta_m-\gamma_m-K_m^L-\delta_m^L\\ -K_m^D-\delta_m^D\\ 0\end{pmatrix}+\eta\begin{pmatrix}p_{x+m}v_L(\eta,m+1,N)+q_{x+m}v_D(\eta,m+1,N)\\ 0\\ 0\end{pmatrix} \tag{8}$$

with

$$\boldsymbol{v}(\eta,N,N)=\begin{pmatrix}P_N-\alpha_N-\beta_N-\gamma_N-K_N^L-\delta_N^L\\ -K_N^D-\delta_N^D\\ 0\end{pmatrix} \tag{9}$$

The first entry of $-\boldsymbol{v}(\eta,m,N)$, denoted $-v_L(\eta,m,N)$, gives total expected discounted value at time m, called in the life insurance literature the life reserve of the product, denoted by Res_m. Such reserve has the property that at the beginning of the policy its value is equal to zero, i.e.:

$$v_L(\eta,0,N)=0. \tag{10}$$

Then we seek premiums $P_0, P_1, \ldots, P_N$ to satisfy relation (10).

If we compute the variance of the total discounted reward we use the fact that

$$\sigma(\eta,N,N)=\begin{pmatrix}0\\0\\0\end{pmatrix} \qquad (11)$$

Hence from relation (3) we can conclude that

$$\left[\sigma(\eta,m,N)\right]_{sq}=\boldsymbol{s}(m)+2\eta\left[\boldsymbol{P}(m)\left[\boldsymbol{v}(\eta,m+1,N)\right]_{diag}\mathbf{R}^{\mathrm{T}}(m)\right]_{diag}\boldsymbol{e}+$$

$$+\eta^2\boldsymbol{P}(m)\left[\boldsymbol{v}(\eta,m+1,N)\right]_{sq}-\boldsymbol{v}(\eta,m,N)_{sq}+\eta^2\boldsymbol{P}(m)\left[\sigma(\eta,m+1,N)\right]_{sq}=$$

$$=\begin{pmatrix}\left[P_m-\alpha_m-\beta_m-\gamma_m-K_m^L-\delta_m^L\right]^2\\ \left[-K_m^D-\delta_m^D\right]^2\\ 0\end{pmatrix}+$$

$$+2\eta\begin{pmatrix}\left[P_m-\alpha_m-\beta_m-\gamma_m-K_m^L-\delta_m^L\right]\left[p_{x+m}v_L(\eta,m+1,N)+q_{x+m}v_D(\eta,m+1,N)\right]\\0\\0\end{pmatrix}+$$

$$+\eta^2\begin{pmatrix}p_{x+m}\left[v_L(\eta,m+1,N)\right]^2+q_{x+m}\left[v_D(\eta,m+1,N)\right]^2\\0\\0\end{pmatrix}-\begin{pmatrix}\left[v_L(\eta,m,N)\right]^2\\ \left[v_D(\eta,m,N)\right]^2\\0\end{pmatrix}+$$

$$+\eta^2\begin{pmatrix}p_{x+m}\left[\sigma_L(\eta,m+1,N)\right]^2+q_{x+m}\left[\sigma_D(\eta,m+1,N)\right]^2\\0\\0\end{pmatrix}$$

Since $\left[v_D(\eta,m,N)\right]^2=\left[K_m^D+\delta_m^D\right]^2$ we can write the following recursive relation for $\left[\sigma(\eta,m,N)\right]_{sq}$

$$\left[\sigma(\eta,m,N)\right]_{sq}=\eta^2\begin{pmatrix}p_{x+m}\left[v_L(\eta,m+1,N)\right]^2+q_{x+m}\left[v_D(\eta,m+1,N)\right]^2\\0\\0\end{pmatrix}-$$

$$-\begin{pmatrix}p_{x+m}v_L(\eta,m+1,N)+q_{x+m}\left[v_D(\eta,m+1,N)\right]^2\\0\\0\end{pmatrix}+\begin{pmatrix}p_{x+m}\left[\sigma_L(\eta,m+1,N)\right]^2\\0\\0\end{pmatrix} \qquad (12)$$

with an initial value (11). Obviously, the variance of the total discounted reward when the policyholder dies or the policy claim is set aside of the portfolio is equal to zero.

4. EXAMPLE

In this section we will illustrate the obtained recursive relation in well-known life insurance product called endowment. We will suppose that the entry age of the policyholder is x, policy term is N and constant sum assured $K > 0$ is paid at the end of the policy (when the death of policy holder occurs or the policy term ends). We consider the case that the premium is paid only at the beginning of the policy term (the product is singly paid). We denote such premium by P.

Obviously, $K_n^D = K, \quad \forall n = 0,1,\ldots,N$ and $K_n^L = 0, \quad \forall n = 0,1,\ldots,N-1, \quad K_n^L = K$.

At first we will suppose that all expenses are equal to zero (we will compute on netto basis). We denote $p_x^{(n)} = \prod_{k=0}^{n-1} p_{x+k}$, $p_x^{(0)} = 1$ and $q_x^{|(n)} = p_x^{(n)} q_{x+n}$, $n \geq 0$. From (8), (9) we compute the reserve $Res_m = -v_L(\eta, m, N)$:

$$Res_N = K,$$

$$Res_{N-1} = \eta K,$$

$$Res_{N-2} = \left(q_{x+N-2}^{|(0)} \eta + p_{x+N-2}^{(1)} \eta^2 \right) K,$$

$$Res_{N-3} = \left(q_{x+N-3}^{|(0)} \eta + q_{x+N-3}^{|(1)} \eta^2 + p_{x+N-3}^{(2)} \eta^3 \right) K,$$

$$\ldots,$$

$$Res_{N-k} = \left(q_{x+N-k}^{|(0)} \eta + \ldots + q_{x+N-k}^{|(k-2)} \eta^{k-1} + p_{x+N-k}^{(k-1)} \eta^k \right) K,$$

$$\ldots,$$

$$Res_0 = \left(q_{x+N}^{|(0)} \eta + \ldots + q_x^{|(N-2)} \eta^{N-1} + p_x^{(N-1)} \eta^N \right) K - P.$$

The single netto premium determines the following equation

$$Res_0 = 0, \tag{13}$$

thus

$$P = \left(q_{x+N}^{|(0)} \eta + \ldots + q_x^{|(N-2)} \eta^{N-1} + p_x^{(N-1)} \eta^N \right) K.$$

The variance is calculated similarly according to (12) and (11) (we compute the variance in state L only since the variance in the other states is equal to zero), recall that $v_D(\eta, m, N) = -K, \quad \forall m = 0,\ldots,N$:

$$[\sigma_L(\eta,N,N)]^2 = 0,$$
$$[\sigma_L(\eta,N-1,N)]^2 = 0,$$
$$[\sigma_L(\eta,N-2,N)]^2 = \left[\left(q_{x+N-2}^{|(0)}\eta^2 + p_{x+N-2}^{(1)}\eta^4\right) - \left(q_{x+N-2}^{|(0)}\eta + p_{x+N-2}^{(1)}\eta^2\right)^2\right]K^2,$$
$$[\sigma_L(\eta,N-3,N)]^2 = \left[\left(q_{x+N-3}^{|(0)}\eta^2 + q_{x+N-3}^{|(1)}\eta^4 + p_{x+N-3}^{(2)}\eta^6\right) -\right.$$
$$\left. -\left(q_{x+N-3}^{|(0)}\eta + q_{x+N-3}^{|(1)}\eta^2 + p_{x+N-3}^{(2)}\eta^3\right)^2\right]K^2,$$
$\dots,$
$$[\sigma_L(\eta,N-k,N)]^2 = \left[\left(q_{x+N-k}^{|(0)}\eta^2 + \dots + q_{x+N-k}^{|(k-2)}\eta^{2(k-1)} + p_{x+N-k}^{(k-1)}\eta^{2k}\right) -\right.$$
$$\left. -\left(q_{x+N-k}^{|(0)}\eta + \dots + q_{x+N-k}^{|(k-2)}\eta^{k-1} + p_{x+N-k}^{(k-1)}\eta^k\right)^2\right]K^2,$$
$\dots,$
$$[\sigma_L(\eta,0,N)]^2 = \left[\left(q_{x+N}^{|(0)}\eta^2 + \dots + q_x^{|(N-2)}\eta^{2(N-1)} + p_x^{(N-1)}\eta^{2N}\right) -\right.$$
$$\left. -\left(q_{x+N}^{|(0)}\eta + \dots + q_x^{(N-2)}\eta^{N-1} + p_x^{(N-1)}\eta^N\right)^2\right]K^2.$$

In what follows we assume the following expenses:

- α expenses: $\alpha_0 = \alpha^{(1)}K + \alpha^{(2)}P,\ \alpha_n = 0,\ \forall n = 1,\dots,N,\ \alpha^{(1)},\alpha^{(2)} \in [0,1]$,
- β expenses: $\beta_n = \beta K,\ \forall n = 0,\dots,N-1,\ \beta_N = 0,\ \beta \in [0,1]$,
- γ expenses: $\gamma_0 = \gamma P,\ \gamma_n = 0,\ \forall n = 1,\dots,N,\ \gamma \in [0,1]$,
- δ^L expenses: $\delta_n^L = 0,\ \forall n = 0,\dots,N-1,\ \delta_N^L = \delta K,\ \delta \in [0,1]$,
- δ^D expenses: $\delta_n^D = \delta K,\ \forall n = 0,\dots,N$.

Then according to (8) and (9) we have:

$$Res_N = (1+\delta)K,$$
$$Res_{N-1} = \beta K + (1+\delta)K,$$
$$Res_{N-2} = \left(1 + p_{x+N-2}^{(1)}\eta\right)\beta K + \left(q_{x+N-2}^{|(0)}\eta + p_{x+N-2}^{(1)}\eta^2\right)(1+\delta)K,$$

$$Res_{N-3} = \left(1 + p^{(1)}_{x+N-3}\eta + p^{(2)}_{x+N-3}\eta^2\right)\beta K +$$
$$+\left(q^{|(0)}_{x+N-3}\eta + q^{|(1)}_{x+N-3}\eta^2 + p^{(2)}_{x+N-3}\eta^3\right)(1+\delta)K,$$

$\dots,$

$$Res_{N-k} = \left(1 + p^{(1)}_{x+N-k}\eta + \dots + p^{(k-1)}_{x+N-k}\eta^{k-1}\right)\beta K +$$
$$+\left(q^{|(0)}_{x+N-k}\eta + \dots + q^{|(k-2)}_{x+N-k}\eta^{k-1} + p^{(k-1)}_{x+N-k}\eta^k\right)(1+\delta)K,$$

$\dots,$

$$Res_0 = \left(1 + p^{(1)}_x\eta + \dots + p^{(N-1)}_x\eta^{N-1}\right)\beta K +$$
$$+\left(q^{|(0)}_x\eta + \dots + q^{|(N-2)}_x\eta^{N-1} + p^{(N-1)}_x\eta^N\right)(1+\delta)K + \left(\alpha^{(1)}K + \alpha^{(2)}P\right) - P(1-\gamma).$$

Again from (13) we have the single premium

$$P = \frac{\left(q^{|(0)}_{x+N}\eta + \dots + q^{|(N-2)}_x\eta^{N-1} + p^{(N-1)}_x\eta^N\right)(1+\delta) + \alpha^{(2)} + \left(1 + \dots + p^{(N-1)}_x\eta^{N-1}\right)\beta}{1 - \alpha^{(2)} - \gamma}K.$$

The corresponding variance can be again obtained from (12) and (11), but the formula has more complex analytic expression and therefore we shall not derive it here.

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ECONOMIC RESEARCH IN THE CZECH REPUBLIC: ENTERING INTERNATIONAL ACADEMIC MARKET[1]

FRANTIŠEK TURNOVEC

Abstract Paper provides a short characteristic of publication outputs of the economists in the Czech Republic during eight years (1993-2000) and presents the project of evaluation of research performance of the Czech economics departments based on records in international databases.

Keywords: citation index, economic research, international databases, research performance

After 1989 economic research in the Czech Republic had faced several difficult problems. The demand for new concepts to implement fast transition from centrally planned economy to a standard market economy led to a rather chaotic discussions on transformation steps, missing traditional academic attributes. It is in some sense understandable: in the hands of the state there was an enormous power and responsibility for unprecedented economic reforms. There was no time for deep analyses and careful evaluation of options. Part of economic community became directly involved in everyday politics on the high positions in governmental offices and in the parliaments. Invasion of outside economic advisors of different rank and quality was a part of the game and supported the impression that a straightforward interpretation of simple neoclassical diagrams is a "scientific way" of solving economic problems. But, from a medium and long run perspective the main problem was to re-establish academic standards in economic research. A characteristic feature of the economic research was an isolation from world standards in presenting research outputs. With a few exception, related mostly to operations research and econometrics, the publication of results was oriented exclusively on Czech (Slovak) economic journals publishing in the Czech language and using not very demanding reviewing procedures. So to overcome isolation and undemanding provincial character of economic science it was necessary to enter international academic market.

The following data about publication activities of the Czech economic community illustrate the process of internationalization of Czech economic research. We are using the data from database of the Government Committee for Research

1 This research was supported by the Grant Agency of the Czech Republic, project No. 402/04/1214.

(GCR), that has an ambition to collect information about all publications of the Czech researchers (not only economists).

By the classification used by the Committee there are two branches of economic science: General Economics (GE) and Applied Statistics and Operations Research (ASOR). The data about journal publications, chapters in books and book monographs in these two branches for the years 1993-2000 were retrieved from this database.[2] Only papers in journals having ISSN registration and chapters in books and book monographs having ISBN registration were considered.

All publications were subdivided by the publishers: out of all publications papers in journals listed in the ECONLIT database[3], and chapters in books and book monographs followed by ECONLIT and Social Science Citation Index (SSCI) were retrieved (classified publications). Within each group additional classification was used by the location of publisher: classified journals published in the Czech Republic and classified journals published in the "West"; [4] classified books published by prestigious Czech academic publishers and books published by the "western" publishers.[5] Another classification was used by the language of publication: publications in English, in other world languages, and in Czech and Slovak as a complement to total (see also Turnovec, 2000). While the GCR database might be incomplete,[6] using the time series provides some general characteristic of the development in the field.

In Table 1, 2 and 3 we provide information (time series 1993-2000) by the types of publications (ISSN journals, ISBN chapters and monographs), separately by GE and ASOR.

Some of the classified international journals in which the economists affiliated in the Czech Republic[7] published at least one paper during this period:

American Economic Review

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2 The GCR and central database of publications were introduced only in 1993, after the separation of Czechoslovakia and establishment of the Czech Republic.

3 ECONLIT is an electronic database of American Economic Association, founded in 1969. It collects records about journal publications, books and research papers. SSCI is an electronic database collecting records about citations of articles, books and other publications in journals oriented on social sciences.

4 There are three classified Czech economic journals, included in JEL database: Politická ekonomie [Political Economy], published by the Prague Economic University, with papers published usually in Czech, Finance a úvěr [Finance and Credit], published by the Charles University in Prague, with papers both in Czech and English language, and Prague Economic Papers, published by the Prague Economic University exclusively in English. By "West" we mean countries of EU, USA and Japan.

5 Only two Czech classified publishers were considered: The Academia Press, publishing house of the Academy of Sciences of the Czech Republic, and The Karolinum Press, publishing house of the Charles University in Prague.

6 The system of updating database is not perfect, the motivation for researchers to keep their records complete is weak and some of publications, related to quantitative economics might be included under mathematical branches of the database, there is also some time lag in recording new publications.

7 I am intentionally not using "Czech economists" to avoid complicated discussion about nationality of authors. There is no "Czech economic science", but "economic science in the Czech Republic".

European Economic Review
Journal of Mathematical Economics
European Journal of Political Economy
Eastern European Economics
International Journal of Finance and Economics
Control and Cybernetics
Communist and Post-communist Studies
Journal of Business Ethics
Economics of Transition
Central European Journal for Operations Research

Table 1, Publications: Papers in ISSN Journals

General Economics	1993	1994	1995	1996	1997	1998	1999	2000
Total	98	193	248	334	600	915	459	450
CZ Politická ekonomie	7	23	30	14	25	38	23	23
CZ - Prague Economic Papers	-	2	8	4	11	14	11	4
CZ - Finance a Úver	10	19	22	12	14	23	13	11
western journals	5	1	1	-	1	17	25	15
Published in English	2	18	28	49	170	88	86	37
Published in other foreign languages	3	-	2	2	2	23	1	8
Applied Statistics and Operations Research								
Total	21	41	43	75	73	180	135	67
CZ Politická ekonomie	1	6	1	3	-	6	6	4
Prague Economic Papers	1	-	-	3	-	-	1	1
Finance a Úver	3	-	2	-	-	2	-	-
western journals	2	1	-	-	3	36	42	6
Published in English	3	11	23	26	39	98	93	36
Published in other foreign languages	-	-	-	-	3	1	-	1

Table 1 demonstrates relatively successful re-orientation of "output flows": while during previous period (1970-1990) there was only one publication in really prestigious economic journal (M. Maòas in Econometrica in 1972) and few publications in specialized journals (e.g. European Journal of Operational Research, Management Science, Zeitschrift fur Operations Research), we can observe a visible presence of economists from the Czech Republic on the international academic market.

We can observe the culmination of the number of journal publication in 1998, what is probably related to the "boom" of transformation topics in the first half of nineties.

The third observation is related to the language of publication: Out of 3932 journal publications 807 were published in English, what represents 20.52%. We can still identify difference between general economics and applied statistics/operations research orientation: while 51,81% of papers in ASOR category were published in English, the corresponding score in GE is only 14.50%.

A survey of thematic orientation of economic articles published by authors from the Czech Republic compared to the rest of Europe see in Machácek, 2004.

Books and chapters in books and monographs are generally considered to be "second best" publications (at least in economic sciences) and mostly they are not considered in evaluation schemes of research. Nevertheless, publications outside of the country are participating in "citation games" and reflects the "export performance" of domestic scientific production. Among prestigious publishers of the texts recorded in this category there are the following international publishing houses:

Academic Press, San Diego
Springer Verlag, Berlin, Heidelberg, New York
Edward Elgar, Aldershot
Kluwer Academic Publishers
Macmillan Press and St. Martin's Press, London, New York
Rowman & Littelfield Publ., Boston

Table 2, Publications: Chapters in ISBN Books

General Economics	1993	1994	1995	1996	1997	1998	1999	2000
Total	-	-	-	6	14	27	99	141
CZ Karolinum	-	-	-	-	-	4	1	2
Academia	-	-	-	-	-	-	-	-
Western publishers	-	-	-	-	-	3	12	7
Published in English	-	-	-	4	-	22	16	24
Published in other foreign languages	-	-	-	-	-	2	2	14

Applied Statistics and Operations Research	1993	1994	1995	1996	1997	1998	1999	2000
Total	-	-	-	-	2	13	13	25
CZ Karolinum	-	-	-	-	-	-	-	-
Academia	-	-	-	-	-	-	-	-
Western publishers	-	-	-	-	-	11	1	3
Published in English	-	-	-	-	-	9	2	11
Published in other foreign languages	-	-	-	-	-	-	1	-

Table 3, Publications: ISBN Book Monographs

General Economics	1993	1994	1995	1996	1997	1998	1999	2000
Total	15	39	28	44	114	94	40	65
CZ Karolinum	-	-	-	-	3	1	-	1
Academia	-	-	-	-	4	-	-	-
Western publishers	4	2	4	2	1	3	2	-
Published in English	-	3	5	1	17	7	1	2
Published in other foreign languages	4	1	-	1	-	1	1	-

Applied Statistics and Operations Research	1993	1994	1995	1996	1997	1998	1999	2000
Total	-	6	1	7	8	9	9	10
CZ Karolinum	-	-	-	-	-	2	-	-
Academia	-	-	-	-	2	-	-	-
Western publishers	-	-	-	1	-	2	2	-
Published in English	-	-	-	1	-	5	5	-
Published in other foreign languages	-	-	-	-	-	-	-	-

For slightly more than a month the Czech Republic together with other nine new member states is sharing the same research area with other European Union countries. Well elaborated methodology of evaluation of research performance of the research institutions (mostly affiliated with universities) is used in the European Union to produce rankings of economic departments in Europe performance (see e.g. Lubrano, Bauwens, Kirman and Protopopescu, 2003). There is no reason to expect that the same standards will not be implemented in evaluation of universities in new member states. To be competitive in Europe and in the world there is still much to be done in changing publication habits.

At present we even do not know where we are in comparative terms: demanding methods of research performance evaluation have not been tested yet in the Czech Republic. The first attempt will take place in 2004 within the project of the Grant Agency of the Czech Republic "Microeconomics of university education and measuring research performance of the universities". The main features of the project are the following:

Objective of the project: to compare objectively measurable research performance of the faculties of economics, institutes and/or departments of economics at non-economic faculties or non-university economic research institutions according to their presence at international academic markets.

Basic principles of evaluation:

a) Minimal demand on cooperation of evaluated institutions.

b) Minimal influence of evaluators on results of evaluation.

c) Use of unquestionable data (international databases).

d) Two components of data: outputs (publications) and response (citations). Data will be weighted by internationally accepted impact factors (Kalaitzidakis, Mamuneas and Stengos, 2001).

e) No other academic activities will be considered.

Units of evaluation:

a) Either faculty of economics, if the faculty performs study programs exclusively in economics,

b) or economics department (institute) at the non-economic faculty,

c) or non-university research unit (e.g. Economics Institute of Academy of Sciences).

Representation of the institution: Only full-time employees in pedagogical and research category. One person can represent only one institution. Only professors and associate professors (senior researchers) will be considered, unless the institution will nominate broader representation.

Primary data:

a) Publication defined as bibliographic record in international databases (Econlit, SCI, Web of Science).

b) Citation defined as citation record in international databases (Econlit, SSCI). Auto-citations are excluded.

c) If the same publication is recorded in different databases, it is considered only once.

d) In the case of co-authorship each of co-authors gets the share 1/n of the publication.

Evaluated period: Last ten years (1994-2003) to get results that can be compared with the results of European rankings. The changes in affiliation of evaluated persons will not be considered, affiliation in 2004 will be significant.

Method of aggregation:

n — number of evaluated institutions (j = 1, 2, …n)

m_j — number of representing persons of the j-th institution (i_j = 1, 2, …, m_j)

p(j, i_j) — number of publication units of the person i_j of institution j, k_{ij} = 1, 2, …., p(j, i_j)

pw(k_{ij}) — share of person i_j from institution j in publication k_{ij} (co-authorship considered)

c(j, i_j) — number of citation units of the person i_j from institution j, t_{ij} = 1, 2, …., p(j, i_j)

cw(t_{ij}) — share of person i_j from institution j in citation t_{ij} (co-authorship considered)

$pf(k_{ij})$ impact-factor of publication k_{ij}, measured by impact-factor of the journal of publication

$cf(t_{ij})$ impact-factor of citation t_{ij}, measured by impact-factor of the journal of citation

r_j total number of employees of the j-th institution in pedagogical/research category

Indicators:

Individual performance of the person i_j from institution j

$$v_{i_j} = \sum_{k_{ij}=1}^{p(j,i_j)} pw(k_{ij})\left(1+pf(k_{ij})\right) + \sum_{t_{ij}=1}^{c(j,i_j)} cw(t_{ij})\left(1+cf(t_{ij})\right)$$

(sum of publication shares weighted by impact-factors increased by 1, plus sum of citation shares measured by impact-factor increased by 1). This measure can be used for ranking of individuals in the Czech Republic (e.g. top 100 economists).

Absolute performance of the institution j

$$V_j = \sum_{i_j=1}^{m_j} v_{i_j}$$

(sum of individual performances of individuals affiliated).

Relative performance of institution j

$$RV_j = \frac{V_j}{r_j}$$

(absolute performance per one employee in pedagogical/research category). Can be used for ranking of institutions.

Relative share of individual i_j in research product of institution j

$$RV_{i_j} = \frac{V_{i_j}}{V_j}$$

(can be used for ranking of individuals within the institution).

Marginal relative contribution of individual i_j to institution j

$$MV_{i_j} = RV_j - \frac{V_j - V_{i_j}}{r_j - 1} = \frac{V_j\left(r_j - 1\right) - \left(V_j - V_{i_j}\right)r_j}{r_j\left(r_j - 1\right)} = \frac{r_j V_{i_j} - V_j}{r_j\left(r_j - 1\right)}$$

(difference between relative performance of the institution j with individual i_j and relative performance of the institution j without individual i_j).

Many other indicators can be constructed and used for statistical analysis. The first version of evaluation will be finalized and released for publication in December 2004.

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