1. Introduction

In this article I would like to make a connection between methods of the operational research and bullwhip effect, which is usually mentioned when talking about supply chain management. Supply chain management (SCM) is nowadays seen as one of the most important areas that should be developed. Supply chain itself can be characterized as a net that consists of suppliers, manufacturing centres, warehouses, distribution centres, and retail outlets, as well as raw materials, work-in-process inventory, and finished products that flow between the facilities. The main effort is dedicated to the interconnection of all subjects in the supply chain and to the optimization of all the flows that exist among these subjects. But because of the fact that firms are used to compete and not to cooperate, they usually don’t want to share any information about their products, inventory, costs or profit. So it is not easy to manage the whole supply chain the way that is acceptable for everyone. I’ve tried to describe some examples connected with inventory optimisation and show, how the effort of minimization of the enterprise inventory cost can cause the bullwhip effect.

2. Bullwhip Effect

Whenever we read any book or an article about the supply chain or the supply chain management, we probably find there something about the bullwhip effect (also called demand amplification effect or whiplash). Forester was the first man who described this effect but the experts of Procter & Gamble gave it its name (after the way the amplitude of a whip increases down its length) and also its publicity. It’s well known, that during the monitoring of the customers demand and retailers and distributors orders of the diaper product “Pampers” they found out the amplification of order variability, though the demand was almost fixed. Recently the interest in the exploration and measurement of the bullwhip effect has grown up because of the fact that the amplification of orders influences the distributor’s and the manufacturer’s costs, inventory, reliability and other important business processes. By reason that nowadays the main effort is dedicated to the coordination and communication among customers, suppliers, distributors and manufacturers (it means it is dedicated to the development of the supply chain management), some specialists of this branch have started to look for the reasons for the bullwhip effect, for the methods of its measurement and also for
the methods of its elimination.

2.1 Four Causes of the Bullwhip Effect

- **Demand forecast.** It’s not assumed that the retailer knows the exact form of the customer demand process. Instead, he uses historical data and some forecasting techniques to estimate the demand. The supplier doesn’t know the retailer’s data and so he supposes the retailer’s order to be the real demand. Due to this, the forecast could be very different and that’s why the orders can vary.

- **Order batching.** In most cases (see below) the cost (or inventory) policy is the main why for ordering in batches. But due to this, the next link of the supply chain has to have higher inventory to avoid the depletion of inventory.

- **Price fluctuation.** Customers are driven to buy in larger quantities by attractive offers on quantity discounts or price discounts. If their behaviour is rational, they buy more when the price is down or less when the price is up. However this strategy doesn’t reflect their true needs, and so this may give birth to the bullwhip effect.

- **Rationing and shortage gaming.** This cause might be similar to the price fluctuation. It occurs when demand exceeds supply or when the customers think it may happen. Then they start to exaggerate their real needs to be sure that the existing demand will be satisfied. The demand amplification effect will grow up even further if customers are allowed to cancel their orders when their real demand is fulfilled.

It’s possible to consider more reasons, for example the information distortion, lead times or the maximization of the profit (or the minimization of the costs) that can influence each of the causes mentioned above.

Every retailer usually knows a lot of information – about customers, demand, prices, discounts, inventory, etc. – but it doesn’t want to share this information with another company even though it might be its supplier. Then the supplier and the manufacturer don’t know much about the real situation on the market, and therefore they can hardly follow the customers’ needs. This effect is called the information distortion.

**Lead times** aren’t conventionally mentioned separately as the main cause of the bullwhip effect, although this factor is included in the bullwhip effect formula. It’s clear that the longer the lead times are, the larger the demand and the safety stock must be (to avoid the inventory shortage), and that’s why the larger the demand amplification could be.

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The most often aim of every company is to maximize their profit or to minimize their costs irrespective of their suppliers or customers. If they try to fulfil this stated goal, they have to realize some of the actions mentioned above (discounts, order batching, etc.) and by this means they contribute to the demand amplification effect.

2.2 Problems With Measuring of the Bullwhip Effect

According to other similar articles, the bullwhip effect is usually measured as the ratio of variances \( \frac{\text{Var}(q^*)}{\text{Var}(Q)} \), where \( \text{Var}(q^*) \) represents variance of (optimal) order and \( \text{Var}(Q) \) is variance of demand faced by the enterprise. Although this formula is used very often, I’ve found out that it is dependent on the type of data aggregation.

I have obtained the real data about the intensity of the sales and orders of one type of summer tyres\(^2\) from the tyre repair and service shop. Chart 1 show the real demand, order and inventory per month.

![Chart 1 – Monthly demand, order and inventory](image)

Demand is between 0 and 47 units (tyres) per month, but the firm orders every month (as you can see later, sometimes even every week or twice a week). The variance of the sales is 241 and the variance of the orders is 210, so there is no demand amplification effect because the quotient of these variances is 0.87.

As I know, the customers don’t buy the tyres once a month but conventionally they want to buy it every week or every day and so does the retailer. When you look at the Chart 2, you can see that the inventory level is much higher than the demand level (they can afford it because of the fact that the inventory holding costs are very small). Although the orders are close to the demand, the bullwhip effect is suddenly 1.34.

\(^2\) Brilantes 160/70/R13 OR60 – usually used for the Czech cars Skoda Fabia and Skoda Felicia
Finally I’ve investigated the daily demand and order (and then also the same but without weekends because the shop is closed in those days). When I’ve taken into account all work days of the year 2001, the bullwhip effect has been 3.27. The question is whether the days with zero demand and zero order should be involved or not. If they aren’t involved, the bullwhip effect is surprisingly 5.84. As you can see in Chart 3, the bullwhip effect also depends on the period of data aggregation.

Another possibility, how to count this effect, is the ratio of coefficients of variation of order and demand, where coefficient of variation is equal to standard deviation divided by average demand (order). This formula is much worse than the previous one, especially when the standard deviations of demand and order are nearly equal, because the ratio of average orders (demands) doesn’t tell us nearly anything about the oscillations.

Because of these difficulties with measuring, I’ve tried to count the mean costs of every member of the supply chain (in the next example) and define the bullwhip effect (with regard to costs) as a percentage growth of costs.
3. Stochastic Models of Inventory and the Bullwhip Effect

It is supposed that demand is a random variable (usually a discrete random variable) that can be approximated with a normal distribution with known mean and standard deviation. If we want to calculate the optimal order quantity (to minimize the inventory costs), we can use the EOQ formula as follows (I assume the inventory shortage doesn’t occur, so this model doesn’t reflect the inventory shortage costs):

EOQ formula: \[ q^* = \sqrt{\frac{2\mu_Q c_2}{c_1}} \]

where: \( q^* \) = the optimal order quantity (EOQ) for minimization of the inventory costs
\( \mu_Q \) = the mean demand in units per one period
\( c_1 \) = the inventory carrying costs (per one unit per one period)
\( c_2 \) = the costs per order

Since we don’t know the actual demand, it is necessary to build up a safety stock (w) that should compensate the variations of demand. The safety stock level depends on the desired percents product availability (\( \alpha \) = the probability that the inventory shortage won’t occur). This desired service level is a function of the normal loss curve, which provides the area in a right tail of a normal distribution. If the factor that corresponds with \( \alpha \) is z and we know the lead times (which are constant), then we can calculate the safety stock level and also the reorder point (the inventory level showing the necessity to order) as:

\[ w = z \cdot \sigma_Q \cdot \sqrt{L} \]
\[ s = L \cdot \mu_Q + w \]

where: \( w \) = safety stock in units, \( z \) = the z factor that corresponds with service level \( \alpha \)
\( L \) = lead time, \( s \) = reorder point in units, \( \mu_Q \) = the mean demand
\( \sigma_Q \) = the standard deviation of demand

Now let’s say that the customers demand oscillates randomly between 20 and 30 units per month (mean is 25 units and standard deviation is 1,67). The fixed costs are 1,5 crowns per month and variable costs are 120 crowns (for all members of the supply chain), the lead times are 2 weeks (0.43478 per month) and the service level is 99%. Then the optimal characteristics for the retailer are in the Table 1. In this case the bullwhip effect is 316 because the oscillations of demand are very small compared to the oscillations of the orders.

Since the demand is random, the supplier can’t find out when the retailer really orders (which week or day). Because the retailer’s order is now its demand, which the supplier faces,
and it is not normally distributed, let’s suppose that the supplier has 3 possible strategies (A, B, C). All characteristics are in Table 1.

Table 1 – Main characteristics of retailer and supplier

<table>
<thead>
<tr>
<th></th>
<th>Retailer</th>
<th>Supplier A</th>
<th>Supplier B</th>
<th>Supplier C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_Q$ (units)</td>
<td>25</td>
<td>21</td>
<td>63</td>
<td>31,5</td>
</tr>
<tr>
<td>$\sigma_Q$ (units)</td>
<td>1,67</td>
<td>30</td>
<td>0,67</td>
<td>10,5</td>
</tr>
<tr>
<td>$q^*$ (units)</td>
<td>63,24</td>
<td>57,96</td>
<td>100,6</td>
<td>71</td>
</tr>
<tr>
<td>$t^*$ (months)</td>
<td>2,53</td>
<td>2,76</td>
<td>1,6 (3-5)</td>
<td>2,25</td>
</tr>
<tr>
<td>$w$ (units)</td>
<td>3</td>
<td>47,59</td>
<td>1,06</td>
<td>16,66</td>
</tr>
<tr>
<td>$s$ (units)</td>
<td>14,63</td>
<td>57,56</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>$S$ (units)</td>
<td>79</td>
<td>116</td>
<td>133</td>
<td>103</td>
</tr>
<tr>
<td>$\mu_N$ (crowns/month)</td>
<td>99,37</td>
<td>158,33</td>
<td>152,19</td>
<td>131,48</td>
</tr>
</tbody>
</table>

As you can see in all examples shown above, when any subject of the supply chain uses the EOQ method for order quantity calculation (to minimize its inventory costs), it always orders in batches after some period and so it should enhance the bullwhip effect. But when I’ve tried to simulate this small supply chain (customer-retailer-supplier) and than count the effect, I’ve found out that the point of view (or the measuring method) is the critical factor. When you look into the table above, you can see that the lower the safety stocks are, the higher the optimal order must be, and vice versa. But the most used formula for counting the demand amplification effect doesn’t care about the safety stocks although they are very important when you are interested in costs and inventory levels. So the bullwhip effect counted as the ratio of variances differs from the ratio of mean costs (see Table 2).

Table 2 – Comparison of the classical BE and the growth of mean costs

<table>
<thead>
<tr>
<th></th>
<th>Retailer vers. Supplier A</th>
<th>Retailer vers. Supplier B</th>
<th>Retailer vers. Supplier C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of variances (BE)</td>
<td>0,959</td>
<td>2,168</td>
<td>1,07</td>
</tr>
<tr>
<td>Ratio of mean costs</td>
<td>1,59</td>
<td>1,53</td>
<td>1,32</td>
</tr>
</tbody>
</table>

In the first case there is no bullwhip effect, because the supplier orders nearly with the same frequency and nearly the same amount as the retailer does, but it doesn’t reflect that the supplier must hold high level of safety stock which increase its inventory holding and
carrying costs. So it is possible to say that the bullwhip effect is hidden in inventory level or in safety stocks.

4. Conclusion

Supply chain management (SCM) is nowadays seen as one of the most important areas that should be developed because the present trend aims at cooperation and managing the whole chain. The bullwhip effect is one of the main problems inside any supply chain because it has bad impact on those enterprises, which are not closed to the end customer. I’ve tried to show how it may come into existence, for example when using the EOQ policy (to minimize the inventory holding and carrying costs), which leads to ordering in batches, and how it is possible to measure it. I’ve found out that the usual method for its measuring shows only the growth of amplification of orders (if occurs), but it doesn’t tell us anything about the impact on the inventory level or inventory costs. If each member of the supply chain wants to minimize its inventory costs, the bullwhip effect always appears. As I’ve described above, sometimes it doesn’t appear as the demand amplification effect, but it is hidden in the growth of the inventory level and the inventory costs. Unfortunately, it is nearly impossible to avoid it, because any company must count with uncertainty.

References


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