INVESTMENT IN HUMANS, TECHNOLOGICAL DIFFUSION AND ECONOMIC GROWTH - AN OPTIMAL CONTROL INTERPRETATION

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Abstract

In the mid 1960s, Nelson and Phelps (1966) proposed an economic growth model in which two factors played a central role on explaining how the physical output evolves over time. These two factors were human capital accumulation (investment in humans) and the dissemination of knowledge (technological diffusion).

In this paper we extend the Nelson-Phelps approach to economic growth by taking into consideration an optimal control problem. It is assumed that the economy has an objective function regarding technology goals, which are essentially two: (i) to expand the theoretical knowledge frontier and (ii) to reduce the gap between ready-to-use techniques and potentially available knowledge. By considering such an objective function, it is straightforward to build an intertemporal optimization setup concerning a two sector scenario. The first sector adapts already available technology to productive uses, while the second is an education sector. In this way, we can study the decisions of economic agents concerning the relation between technology and human capital accumulation under an intertemporal perspective.

1. Introduction

In most economic growth models human capital and technology alternatively arise as the main engines of growth. They acquire such crucial property when introduced into an aggregate production function alongside with physical capital and labor. Solow (1956) and Swan (1956) have demonstrated that under a neoclassical production function, these two former inputs are unable to produce, only by themselves, long run sustained growth. Uzawa (1965), Lucas (1988), Caballé and Santos (1993), Mulligan and Sala-i-Martin (1993), Xie (1994), Bond, Wang and Yip (1996) and Ladrón-de-Guevara, Ortigueira and Santos (1999), among others, have modeled human capital formation and they have included this type of input in the aggregate production function in order to support the concept of endogenous growth. Romer (1986, 1990), Aghion and Howitt (1992), Jones (1995), Evans, Honkapohja and Romer (1998) and Young (1998)

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have chosen to put technology in the center of economic growth explanations, and a time dependent technological index arises in the production function as the vehicle to sustained growth.

The papers above mentioned, as well as the whole discussion around aggregate growth models, put the various inputs of production at a same level of analysis. This means that in both the physical capital/human capital approach and the physical capital/technology approach all the inputs that are relevant to growth have to appear as arguments in the final goods production function. As an early attempt to the interpretation of economic growth, Nelson and Phelps (1966) avoid such a straightforward view. For them, it makes sense to assume that human capital serves as a means to generate and spread technology and that technology is then usable in the physical goods production function. Recognizing that the Solow-Swan paradigm is valid when assuming a fixed level of technology, we may continue to use such a framework but should replace the constant level of technology by a time dependent variable, where the respective time path is determined by two factors: investment in humans and technological diffusion.

In this paper the Nelson-Phelps approach is revisited. We study growth giving particular attention to technology and human capital and attributing a subsidiary role to the capital accumulation constraint of the Solow/Swan model and to the consumption utility intertemporal maximization framework developed by Ramsey (1928), Cass (1964) and Koopmans (1964). Our main argument is that, since human capital and technology are the variables that determine the nature of growth, the basic growth results can be derived from the analysis of the factors that influence the time evolution of these variables. The Solow/Ramsey framework becomes accessory and may be used solely to justify that the main economic aggregates (per capita output, physical capital per labor unit, per capita consumption) follow a same long run growth rate as that of the technology variable.

Modeling human capital in a different way from those usually found in the literature on economic growth will allow for two new results. First, we will be able to define a human capital production function where decreasing returns are compatible with long run constant per capita economic growth, what implies that a counterfactual aspect of two sector growth models can be removed from such models: the linearity problem in aggregate capital as raised by Solow (1994). We do not have to assume any kind of artificial knife-edge linearity to encounter a constant long run growth rate. The second result will appear, at least at a first glance, a more awkward one. The way in which we model human capital will lead to a long run steady state
where all relevant per capita variables (physical capital, output, consumption, technology) grow at a same rate while human capital will exhibit a zero growth rate in the long run solution. Growth models generally present human capital as growing alongside with the other economic variables. Here, we obtain the result that there is convergence of the human capital variable to a constant long run value. This may be supported by the idea that we may expand techniques and physical goods indefinitely but there is a limit to the expansion of human capabilities.

In what concerns the modeling of technology we follow the Nelson-Phelps approach by distinguishing two concepts of technology. First, there is a theoretical level of technology or a technological possibilities frontier; second, we assume the existence of a level of technology in practice which is a fraction of the former and can directly be applied to productive uses when embodied in an aggregate final goods production function. The treatment that will be given to technology choices implies the consideration of an optimal control problem in which the goals are simultaneously to amplify the frontier of technological knowledge and to approximate the level of ready-to-use technology to that frontier. This optimization problem is constrained precisely by the Nelson-Phelps motion equation that characterizes the relation between the two technology concepts.

The remainder of the paper is organized as follows. Section 2 develops the structure of the model, section 3 solves the model and refers to the steady state results and section 4 concludes.

2. A Two Sector Model of Optimal Technology Choices

Take technology and human capital as the engines for economic growth. The first variable, technology, may be decomposed into two parts. The technology possibilities frontier will be variable $T(t)$ and a ready-to-use in production technology is represented by $A(t)$. In every time moment $A(t) \leq T(t)$; both variables are assumed to be positive quantities for all $t \geq 0$. We will be concerned with understanding the temporal evolution of a gap variable $\phi(t) = A(t) / T(t)$, $\phi(t) \in [0,1]$, that has a straightforward interpretation: the higher the value of $\phi(t)$ the smaller is the gap between the values of the technology variable representing knowledge immediately available to produce and of the technology variable representing the scientific state of the art. Another fundamental variable is the growth rate of the technology frontier: $\tau(t) = \dot{T}(t) / T(t)$, $T(0) = T_0$ given. In our model this growth rate is assumed as a control variable for the representative agent; that is, in what concerns technology decisions, the economy is able to choose the rate at which scientific progress occurs. Nevertheless, obviously this must be a constrained decision process because in order to allocate more resources to research activities one faces an opportunity cost related to the resources that remain available.
for other economic activities. In particular, in our framework we assume that the choice in terms of basic technology progress is constrained by the necessity of using economic resources to apply technology to the goods production process. Therefore, a trade-off emerges between our first two endogenous variables: $\phi(t)$ and $\tau(t)$.

As far as technology choices are concerned, the basic economic goal is to maximize the intertemporal stream of $v[\phi(t), \tau(t)]$ functions, being a function $v$ defined as follows.

**Definition 1. Function $v$.** The representative agent that makes technology choices faces a real valued objective function $v: \mathbb{R}^2_+ \mapsto \mathbb{R}$ that obeys the following properties:

i) Continuity, concavity and smoothness

ii) $v_{\phi} \frac{\phi(t)}{v} = \theta \in (0,1); \quad v_{\tau} \frac{\tau(t)}{v} = \mu \in (0,1)$

The second condition in definition 1 ensures that the utility of reducing the technology gap and the utility of increasing the pace of technological progress are both positive and diminishing for the representative agent. To simplify our treatment of the model we will work with a specific functional form of the above defined $v$ function.

$$v[\phi(t), \tau(t)] = \phi(t)^{1/(1+\sigma)} \cdot \tau(t)^{\sigma/(1+\sigma)}$$

(1)

The correspondence between parameters $\theta$ and $\mu$ in definition 1 and parameter $\sigma$ in equation (1) is the following: $\sigma = \mu / \theta$ and $\mu = 1 - \theta$.

The trade-off between the two endogenous technology variables, $\phi(t)$ and $\tau(t)$, becomes explicit by considering the Nelson-Phelps technology constraint:

$$\dot{A}(t) = g[h(t)], [T(t) - A(t)], \quad g(0) = 0, \quad g' > 0, \quad g'' < 0, \quad A(0) = A_0 \text{ given}$$

(2)

Eq. (2) states that the rate of increase of the index $A(t)$ is a function of human capital $[h(t)$ is a human capital per unit of labor variable or a human capital efficiency index] and of the gap that exists between the two technology variables. First and second derivatives of $g$ indicate that there are positive but diminishing returns of human capital in the production of technology. The gap term translates the idea that the level of technology in practice will evolve faster when there is a large gap between technology possibilities and the stock of knowledge instantly available to produce.

Recovering variable $\phi(t)$, from equation (2) we arrive at the final form of the first resource constraint of the optimal control problem:

$$\dot{\phi}(t) = g[h(t)], [1 - \phi(t)] - \tau(t). \phi(t), \quad \phi(0) = \phi_0$$

(3)
To complete the presentation of the model it is necessary to define a rule for the evolution of the human capital variable. On this respect we follow the standard form in most growth models, i.e., human capital evolves in time through a production process that involves a production function, $f$, and a constant depreciation rate, $\delta$.

$$\dot{h}(t) = f[h(t)] - \delta \cdot h(t), \quad \delta > 0, \quad h(0) = h_0 \text{ given}$$  \hspace{1cm} (4)

The human capital variable is, alongside with variable $\phi(t)$, a state variable of the intertemporal control problem. Equations (3) and (4) are the motion equations that constitute the resource constraints to which the optimization problem is subject to.

The analytical tractability of the model demands that we take explicit functional forms for functions $f[h(t)]$ and $g[h(t)]$. Assuming that it is possible to choose at each moment in time the shares of human capital to allocate to each of the two economic sectors (technology and education sectors), we define a new variable $u(t)$ that represents precisely the share of human capital allocated to the generation of technology. Obviously $u(t) \leq 1, \quad \forall t$. The properties of function $g$ were set forth in equation (2). Positive and diminishing returns of human capital in the production of technology imply a function with the following shape:

$$g[h(t)] = a \cdot [u(t) \cdot h(t)]^\eta, \quad a > 0, \quad \eta \in (0,1)$$  \hspace{1cm} (5)

Function $f$ may be defined in a similar way:

$$f[h(t)] = b \cdot [(1-u(t)) \cdot h(t)]^\beta, \quad b > 0$$  \hspace{1cm} (6)

For equation (6) we find it convenient to impose $\beta \in (0,1)$ in order to obtain a long run balanced growth path. Thus, one assumes that the education sector exhibits diminishing returns in the accumulation of human knowledge. This point was referred to earlier in the introduction and should be stressed since it constitutes an innovation relatively to conventional growth models. In our model, where human capital contributes to the generation of physical goods solely in an indirect manner, we must assume diminishing returns in the accumulation of human capital in order to get a long term constant steady state growth. As a result of this assumption, the human capital per unit of labor variable will have a different behavior from those of the other inputs: its level will not grow in the long run in opposition to what happens to the several per capita variables, namely output, consumption and physical capital. Remembering that $h(t)$ defines average individual skills, intuitively it is hard to support the concept that human capabilities may be improved further and further indefinitely at a constant rate. Technology and machines can be expanded without limit, individual skills cannot - this is a fundamental argument of our analysis.
The optimal control growth problem has now all the necessary ingredients. Definition 2 states the contours of the optimal solution.

**Definition 2. Control problem optimal solution.** An optimal solution is a set of paths \( \{\phi(t), h(t), \tau(t), u(t)\} \) that solve the maximization problem

\[
\text{Max}_{\tau(t),u(t)} \int_{0}^{\infty} v[\phi(t), \tau(t)] e^{-\rho t} dt
\]

subject to constraints (3) and (4) and where functions \( v, f \) and \( g \) are defined respectively by (1), (5) and (6). All variables assume non-negative values, initial values for state variables are given and shares \( \phi(t) \) and \( u(t) \) remain always below unity. Furthermore, it is obvious from the problem that we take an infinite horizon and that future technology accomplishments are discounted at a constant discount rate, \( \rho > 0 \).

### 3. Optimal Solution and Steady State

Let \( p_\phi(t) \) and \( p_h(t) \) be co-state variables. We synthesize our model's information into a current value Hamiltonian function:

\[
\mathcal{H} = \equiv v[\phi(t), \tau(t)] + \{a. [u(t). h(t)]^\eta [1 - \phi(t)] - \tau(t) . \phi(t)\} . p_\phi(t) + \\
+ (b. ([1 - u(t)]. h(t))^\beta - \delta . h(t)) . p_h(t)
\]

Applying the Pontryagin's maximum principle, the first order optimality conditions can be computed:

\[
\mathcal{H}_\tau = 0 \Rightarrow \frac{\sigma}{1+\sigma} \left[ \frac{\phi(t)}{\tau(t)} \right]^\gamma_{\tau,\sigma} = p_\phi(t) . \phi(t)
\]

\[
\mathcal{H}_u = 0 \Rightarrow \eta . a . u(t)^{-\gamma_{\tau,\sigma}} . h(t)^\eta . [1 - \phi(t)] . p_\phi(t) = \\
\beta . b . [1 - u(t)]^{-\gamma_{\tau,\sigma}} . h(t)^\beta . p_h(t)
\]

\[
\mathcal{H}_\phi = \rho . p_\phi(t) - \dot{p}_\phi(t) \Rightarrow \\
\dot{p}_\phi(t) = \{ \rho + a . [u(t). h(t)]^\eta + \tau(t) \} . p_\phi(t) - \frac{1}{1+\sigma} \left[ \frac{\tau(t)}{\phi(t)} \right]^\gamma_{\tau,\sigma}
\]

\[
\mathcal{H}_h = \rho . p_h(t) - \dot{p}_h(t) \Rightarrow \\
\dot{p}_h(t) = \{ \rho + \delta - \beta . b . [1 - u(t)]^\beta . h(t)^{-\gamma_{\tau,\sigma}} \} . p_h(t) - \\
- \eta . a . u(t)^\eta . h(t)^{-\gamma_{\tau,\sigma}} . [1 - \phi(t)] . p_\phi(t)
\]

\[
\mathcal{H}_{p_\phi} = \dot{\phi}(t) \Rightarrow \dot{\phi}(t) = a . [u(t). h(t)]^\eta . [1 - \phi(t)] - \tau(t) . \phi(t)
\]

\[
\mathcal{H}_{p_h} = \dot{h}(t) \Rightarrow \dot{h}(t) = b . ([1 - u(t)]. h(t))^\beta - \delta . h(t)
\]
\[
\lim_{t \to +\infty} p_\phi(t).e^{-\rho .t}.\phi(t) = 0 \quad (14)
\]
\[
\lim_{t \to +\infty} p_h(t).e^{-\rho .t}.h(t) = 0 \quad (15)
\]

Conditions (8)-(15) are sufficient conditions for optimality, given that the optimal Hamiltonian is concave in \([\phi(t), h(t)]\). The optimality conditions are the relations necessary to prove the following proposition.

**Proposition 1. Existence and uniqueness of a balanced growth equilibrium.** Under the condition \(\beta + p/\delta < 1\) there exists a unique balanced growth path or unique steady state four dimensional point that satisfies (8) to (15).

**Proof:** To prove the existence of a unique steady state point one has to solve the system \[
[\phi(t), h(t), \tau(t), u(t) = \vec{0}].
\]
The solution for this system consists on a set \(\{\phi, h, \tau, u\}\) of constant values. To solve the system one is compelled to find equations of motion for the two control variables. Let us start with \(\tau(t)\).

Differentiating (8) with respect to time, the following relation involving growth rates is obtained: 1

\[
\gamma_\tau = -\sigma \gamma_\phi - (1 + \sigma) \gamma_p \quad (16)
\]

Replacing \(\gamma_p\) and \(\gamma_\phi\) in (16) by the corresponding expressions from (10) and (12), the motion equation for \(\tau(t)\) comes as

\[
\dot{\tau}(t) = \left\{ \frac{1}{\sigma}.\tau(t) - (1 + \sigma).\rho - a.\left[u(t).h(t)\right]^\eta.\left[1 + \sigma/\phi(t)\right] \right\}.\tau(t) \quad (17)
\]

The time evolution of \(u(t)\) is derived from the following relation that is true under (9):

\[
\gamma_u = \frac{1 - u(t)}{(1 - \eta) - (\beta - \eta).u(t)} \left[ \gamma_p - \gamma_p - \frac{\phi(t)}{1 - \phi(t)}.\gamma_\phi + (\eta - \beta)\gamma_h \right] \quad (18)
\]

Finally, we can arrive at the result

\[
\dot{u}(t) = \frac{1 - u(t)}{(1 - \eta) - (\beta - \eta).u(t)} \left[ \frac{\phi(t)}{1 - \phi(t)} \left[ \frac{1 - \sigma}{\sigma} \right].\tau(t) + 
+ [\eta + (\beta - \eta).u(t)].b.\left[(1 - u(t)).h(t)\right]^{(1 - \beta)} - (1 - \beta + \eta).\delta \right].u(t)
\]

1 The symbol \(\gamma\) represents the growth rate of the variable refered in index.
Equations (17) and (19), alongside with the two resource constraints, constitute the system from which we derive the steady state solution. The system has in fact a unique solution, which is

\[
\begin{bmatrix}
\frac{(1-\sigma)b}{\sigma(\rho+\bar{g})} \\
\frac{b}{\delta}(\beta+\rho/\delta)^{\beta} \\
\frac{\sigma}{1-\sigma}(\rho+\bar{g}) \\
1-\beta-\rho/\delta
\end{bmatrix}^{1/(1-\beta)}
\]

with \(\bar{g} = a.(\bar{u},\bar{h})^\eta\). If \(\beta+\rho/\delta<1\) then we guarantee that \(\bar{u} \in (0,1)\) and this is the only boundary condition that must be imposed in order to have (20) as a feasible four dimensional steady state point.

The steady state results deserve some comments. First, we notice that the share \(\bar{u}\) depends upon parameters \(\beta, \rho\) and \(\delta\). The higher the elasticity parameter \(\beta\) and the discount rate, the lower is the value of the share of human capital allocated to technological development. The faster the depreciation of human capital, the more this form of capital is allocated to its own production relatively to a technological use. Second, the human capital efficiency index is indeed a constant amount on the balanced growth path. It depends on parameters \(b, \delta, \rho\) and \(\beta\). Third, we observe that \(\bar{\phi}\) obeys the boundary condition \(\bar{\phi} \leq 1\), because \(\rho>0\) and \(\bar{g}>0\). Fourth, the technology growth rate arises in the steady state depending on a multiplicity of factors, namely (i) the objective function parameter, (ii) the intertemporal discount rate, (iii) both sectors education functions parameters (\(a\) and \(b\)), (iv) the elasticity parameters (\(\beta\) and \(\eta\)) and (v) the depreciation rate.

For the steady state growth rate result,

\[
\tau = \frac{\sigma}{1-\sigma}\left(\rho + a.\left\{(1-\beta) - \rho/\delta\right\}.\left\{(\beta + \rho/\delta)^{\beta}\right\}^{1/(1-\beta)}\right)^{\eta}
\]

we stress that the partial derivatives \(\partial \tau/\partial \sigma > 0\), \(\partial \tau/\partial \bar{u} > 0\), \(\partial \tau/\partial b > 0\) and \(\partial \tau/\partial \eta > 0\) have unquestionable signs. The same is not true for \(\beta, \rho\) and \(\delta\) because when changes in these factors benefit the accumulation of human knowledge (\(h\) rises) they affect negatively the transference of human capital to innovation purposes (\(u\) falls).
At this stage, after the long term technology growth rate has been found, we could transform the model by adding a physical capital accumulation constraint that should include a labor augmenting aggregate production function; which would lead to an economic growth framework. Due to lack of space, this framework will not be developed here but the results of its consideration can be stated in a brief manner. Under such a scenario the growth rate in (21) would be the steady state economic growth rate for the various per capita aggregates: physical capital, consumption and output. In such a model we have endogenous growth through technology choices, that is, long run constant per capita growth is obtained and is a function of several parameters of our analysis. These parameters concern the way in which the economy is able to create and diffuse technology and to invest in human capabilities. In fact, the growth rate obtained in this new framework is precisely the result of the particular way the economy handles the two true engines of growth: human capital and technology.

4. Final Remarks

This paper deals with the same issues as that of Nelson and Phelps (1966). We try to understand how physical capital accumulation plays a subsidiary role in terms of long run growth and the analysis focus on the two central engines of growth: investment in humans and technological diffusion. The way in which technological diffusion is modeled also relies on the Nelson-Phelps approach: the technology index to be included in an aggregate production function evolves over time according to a technological gap that relates to a reference value that may be understood as the scientific frontier or the state of the art in terms of knowledge capabilities of the economy.

The major modeling innovation in our paper consists of the assumption that the economy has the ability to control the growth of the knowledge frontier. This assumption makes sense if one considers a trade-off, arising from the technology resource constraint, between the creation of knowledge and the application of knowledge in material production. Therefore, the control of decisions about technology creation is conditioned by the economic non controllable rules of technology adoption. The way we have chosen to study simultaneously the behavior of human capital and technology variables leads to a two-sector optimal control problem, from which we have obtained some meaningful results:

(i) In the steady state, the state variables ("technology gap" and "human capital efficiency index") and the control variables ("growth rate" and "human capital share") they all display constant values.
(ii) The constant steady state result is possible only if one assumes decreasing returns in human capital accumulation in both the technology and the education sectors. In this way decreasing returns become compatible with sustained growth.

Finally, to translate our analysis to an economic growth setup it is just necessary to include a capital accumulation constraint and, eventually, an intertemporal consumption utility optimization framework. In this way, the rate that describes the evolution over time of the technology variables would simultaneously be the rate of economic growth, under a steady state perspective. Thus, in such a framework, it would be possible to talk about endogenous economic growth because the growth rate would be determined endogenously and because the several parameters that appear in its long run expression may be influenced by the economic agents decisions, including the policy measures that the government undertakes. Nevertheless, the endogenous nature of our growth model is different from the nature of conventional growth models. First, only technology and education decisions influence long run growth; factors as the consumption impatience, savings decisions or the rate of population growth are absent from the steady state growth rate expression. Second, despite the fact that the output, the physical capital stock and the technology all grow in the steady state, the human capital average efficiency does not, which implies that the model is somewhat pessimistic about human capabilities under our assumptions.

References


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