The economic time series of unemployment are frequently affected by policy changes and others events that are known to have occurred at a particular point of time. Events of this type, whose timing are known, have been termed interventions by Box and Tiao (1975). Interventions can affect a time series in a several ways. They can change the level, either abruptly or after some delay, change trend, or lead to other, more complicated, response pattern. Ignoring interventions can lead to an inadequate ARIMA model being fitted and a poor forecast being made.

Interventions can be incorporated into univariate ARIMA model by extending it to include deterministic (or dummy) input variables $I_t$. $I_t$ is the dummy or indicator sequence taking values 1 and 0 to denote the occurrence or nonoccurrence of the exogenous intervention. The following dummy variables have been found to be useful for representing various forms of interventions:

1) A *pulse* variable, which models an intervention lasting only for the observation T,

$$I_t = \xi_t^{(T)}, \text{ where } \xi_t^{(T)} = \begin{cases} 1, & t = T \\ 0, & t \neq T. \end{cases}$$

2) A *step* variable, which models a step change in $y_t$ beginning at observation T,

$$I_t = \xi_t^{(T)}, \text{ where } \xi_t^{(T)} = \begin{cases} 1, & t < T \\ 0, & t \geq T. \end{cases}$$

3) An *extended pulse* variable, useful for modeling ’policy on-policy off’ interventions,

$$I_t = \eta_t^{(T_1, T_2)}, \text{ where } \eta_t^{(T_1, T_2)} = \begin{cases} 1, & T_1 \leq t \leq T_2 \\ 0, & \text{otherwise}. \end{cases}$$
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noting that \( \eta_{(T_1,T_2)} = \sum_{j=0}^{T_2-T_1} \varepsilon(T_1+j) \), \( y_t = (1 + B + \ldots + B^{T_2-T_1}) s_j^{(T_2)} \).

If \((T_1,T_2)\) is generated by an ARIMA(p,q) process, then an intervention model may be postulated as

\[ y_t = \nu(B)I_t + N_t \]

where

\[ N_t = \frac{\theta(B)}{\phi(B)} a_t \]

is the `noise` model, \( \nu(B) \) is a (possibly infinite) polynomial which may admit a rational form such as

\[ \nu(B) = \frac{\omega(B)}{\delta(B)} B^b, \]

where

\[ \omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \ldots - \omega_m B^m, \]

\[ \delta(B) = 1 - \delta_1 B - \ldots - \delta_l B^l, \]

where \( b \) measures the delay in effect (or dead time).

In general, if polynomial \( \delta(B) = 1 \) \((r = 0)\), finite responses of length \( m \) are obtained, whereas if \( r > 0 \), responses of infinite length are obtained.

For various specifications of \( \nu(B) \) we have various responses to pulse and a step change `input` that are of practical interest.

1) If time series \( y_t \) is affected by a pulse input \( I_t = \varepsilon^{(T)} \), the polynomial \( \nu(B) \) has several forms:

a) \( \nu(B) = \frac{\omega}{1 - \delta B} \),

what means, that \( I_t \) has only a transient effect on \( y_t \) with \( \omega \) measuring the initial increase and \( \delta \) the rate of decline. If \( \delta = 0 \), then only an instantaneous effect is felt, whereas if \( \delta = 1 \), the pulse input is really a step change, and the effect is permanent;
b) \( v(B) = \frac{\omega_0}{1 - \delta B} + \frac{\omega_1}{1 - B} \),

represents the situation where, apart from the transient effect \( \omega_0 \), the possibility is entertained that a permanent gain (or loss), \( \omega_1 \) in \( y_t \) is obtained;

c) \( v(B) = \omega_0 + \frac{\omega_1}{1 - \delta B} + \frac{\omega_2}{1 - B} \),

shows the case of an immediate positive response followed by a decay and, possibly, a permanent residual effect, and this might well represent the dynamic response of unemployment to a policy decision.

2) If time series \( y_t \) is affected by a step input \( I_t = \xi_t^{(7)} \), the polynomial \( v(B) \) has also several forms:

a) \( v(B) = \omega \),
shows an immediate step response of \( \omega \);

b) \( v(B) = \frac{\omega}{1 - \delta B} \),

shows the situation of first-order dynamic response and an eventual, or long-run response, of \( \frac{\omega}{1 - \delta} \);

c) \( v(B) = \frac{\omega}{1 - B} \),

represents the case when \( \delta = 1 \), in which the step change produces a ramp or trend in \( y_t \).

Obviously, these models can be readily extended to represent many situations of potential interest.

If the noise model is of multiplicative form

\[
N_t = \frac{\theta(B)\Theta(B')}{(1 - B)^d(1 - B')^\phi(B)\Phi(B')} a_t, \quad (4)
\]

and if there are \( J \) interventions, the model given by equations (1)-(3) can be extended to
\[(1 - B)^d (1 - B^s)^{d_0} y_t = \sum_{j=1}^{J} \frac{\omega_j(B)}{\delta_j(B)} B^{h_j} (1 - B)^d (1 - B^s)^{D_j} I_{j} + \frac{\Theta(B) \Theta(B^s)}{\phi(B) \Phi(B^s)} a_t. \quad (5)\]

In building models of the type (5), parsimonious forms are initially postulated to represent the expected effects of the interventions, with more complex form only being considered if knowledge of the dynamics of the intervention, or subsequent empirical evidence, suggests so. The identification procedures may be applied to data prior to the occurrence of the interventions if a sufficiently large number of such observations are available. If the effects of interventions are expected to be transient, then the identification procedures may be applied to the entire data set. Alternatively, the response polynomials \( \nu_j(B) \equiv \delta_j^{-1}(B) \omega_j(B) B^{h_j} \) may be estimated by least squares method, for suitably large maximum orders, and the identification procedures applied to the residuals \( y_t - \sum \hat{\nu}_j(B) I_{jt} \).

Once a model (5) has been specified, the intervention and noise parameters can be estimated simultaneously by maximum likelihood or nonlinear least squares.

The mentioned theory of intervention models will be applied to the variables of unemployment of school-leavers to express the effects of policy decisions of the government.

References

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